Survey of Modal Techniques Applicable to Autonomous/Semi-Autonomous Parameter Identification

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ABSTRACT

Traditionally, modal analysis has been the purview of the expert. However, as the technology has matured, analysis is increasingly being performed by less experienced engineers or technicians. To address this development, software solutions are frequently focusing upon either wizard-based or autonomous/semi-autonomous approaches. In this paper, several techniques applicable to either approach are presented. By combining various traditional modal parameter estimation algorithms with a-priori constraint/decision information, the process of identifying the modal parameters (frequency, damping, mode shape, and modal scaling) can be greatly simplified and/or automated. Two different methods are described and examples of the efficacy of these techniques are shown for both laboratory and real-world applications.

Nomenclature

N_i = Number of inputs.
N_o = Number of outputs.
N_S = Short dimension size.
N_L = Long dimension size
N = Number of modal frequencies.
λ_r = S domain polynomial root.
λ_r = Complex modal frequency (rad/sec).
$\lambda_r = \sigma_r + j\,\omega_r$
σ_r = Modal damping.
ω_r = Damped natural frequency.
z_r = Z domain polynomial root.
$\{\psi_r\}$ = Base vector (modal vector).
$\{\phi_r\}$ = Pole weighted base vector (state vector).
$[A_r]$ = Residue matrix, mode r.
$[I]$ = Identity matrix.
t_i = Discrete time (sec).
ω_i = Discrete frequency (rad/sec).
s_i = Generalized frequency variable.
$x(t_i)$ = Response function vector ($N_o \times 1$)).
$X(\omega_i)$ = Response function vector ($N_o \times 1$)).
$f(t_i)$ = Input function vector ($N_i \times 1$))
$F(\omega_i)$ = Input function vector ($N_i \times 1$)).
$[h(t_i)]$ = IRF matrix ($N_o \times N_i$)).
$[H(\omega_i)]$ = FRF matrix ($N_o \times N_i$)).
$[\alpha]$ = Numerator polynomial matrix coefficient.
$[\beta]$ = Denominator polynomial matrix coefficient.
m = Model order for denominator polynomial.
n = Model order for numerator polynomial.
v = Model order for base vector.
r = Mode number.

1. Introduction

The desire to estimate modal parameters automatically, once a set or multiple sets of test data are acquired, has been a subject of great interest for more than 40 years. In the 1960s, when modal testing was limited to analog test methods, several researchers were exploring the idea of an automated test procedure for determining modal parameters [1-3]. Today, with the increased memory and compute power of current computers used to process test data, an automated or autonomous, modal parameter estimation procedure is entirely possible and is being attempted by numerous researchers.

Before proceeding with a discussion of autonomous modal parameter estimation, some philosophy and definitions regarding what is considered autonomous is required. In general, autonomous modal parameter estimation refers to an automated procedure that is applied to a modal parameter estimation algorithm so that no user interaction is required once the process is initiated. This typically involves setting a number of parameters or thresholds that are used to guide the process in order to exclude solutions that are not acceptable to the user. When the procedure finishes, a set of modal parameters is identified that can then be reduced or expanded if necessary. The goal is that no further reduction, expansion or interaction with the process will be required.

The larger question concerning autonomous modal parameter estimation is the intended user. Is the autonomous modal parameter estimation procedure expected to give results sufficiently robust for the novice user? This implies that the user could have no experience with modal analysis and, therefore, have no experiential judgement to use in assessing the quality of the results. The use of the term *wizard* implies that this is the desired situation. In contrast, the user could be very knowledgeable in the theory and experienced. For this case, the autonomous modal parameter estimation procedure is simply an efficient mechanism for sorting a very large number of solutions into a final set of solutions that satisfies a set of criteria and thresholds that are acceptable to the user. This user is the assumed reader for the purposes of this discussion.

In order to discuss autonomous modal parameter estimation, some background is needed to clarify terminology and methodology. First, a brief overview of modal parameter estimation methods will be presented. Second, this is followed by a brief overview of documented autonomous modal parameter estimation methods. Third, an overview of some of the tools or semi-autonomous methods that are useful as part of the autonomous modal parameter estimation procedure are reviewed. Finally, the procedures for two autonomous modal parameter estimation methods are presented followed by a number of examples.

2. Background: Modal Parameter Estimation

All modern, commercial algorithms for estimating modal parameters from experimental input-output data can be developed or explained in terms of polynomial based models. For this reason, with minor implementation differences, all of these algorithms can take advantage of the consistency diagram as an aid in identifying the correct modal frequencies from the large number of poles that are found. This section quickly overviews the development of the polynomial models for both the time or frequency domains so that the model order variation options, that are involved in the consistency diagram, can be discussed. This background is detailed more fully in several references [4-8]. The algorithms that commonly use an implementation of the consistency diagram for identifying modal parameters are summarized in Table 1.

2.1 Polynomial Modal Identification Models

Rather than using a physically based mathematical model, the common characteristics of different modal parameter estimation algorithms can be more readily identified by using a matrix coefficient polynomial model. One way of understanding the basis of this model can be developed from the polynomial model used historically for the frequency response function. Note the nomenclature in the following equations regarding measured frequency ω_i versus generalized frequency s_i. Measured input and response data are always functions of measured frequency but the generalized frequency variable used in the model may be altered to improve the numerical conditioning as is done with most frequency domain methods (normalized frequency) and specifically with the polyreference least squares complex frequency (PLSCF) method (complex Z transform of frequency). The commercial implementation of the PLSCF method is known as PolyMAX ®.

Therefore, the multiple input, multiple output (MIMO) FRF model is:

$$\sum_{k=0}^{m} \left[[\alpha_k] (s_i)^k \right] \left[H(s_i) \right] = \sum_{k=0}^{n} \left[[\beta_k] (s_i)^k \right] [I] \tag{1}$$

Equation (1) is evaluated at many frequencies (ω_i) until all data are utilized or a sufficient overdetermination factor is achieved. Note that both positive and negative frequencies are required in order to accurately estimate conjugate modal frequencies. This allows for the coefficients of a matrix coefficient, characteristic polynomial to be identified for a given model order m. The roots of this polynomial can be used to find the modal parameters.

For the general multiple input, multiple output case:

$$\sum_{k=0}^{m} [\alpha_k] \{x(t_{i+k})\} = \sum_{k=0}^{n} [\beta_k] \{f(t_{i+k})\} \tag{2}$$

If the discussion is limited to the use of free decay or impulse response function data, the previous time domain equations can be simplified by noting that the forcing function can be assumed to be zero for all time greater than zero. If this is the case, the $[\beta_k]$ coefficients can be eliminated from the equations.

$$\sum_{k=0}^{m} [\alpha_k] \left[h(t_{i+k}) \right] = 0 \tag{3}$$

Additional equations can be developed by repeating Equation (3) at different time shifts into the data (t_i) until all data are utilized or a sufficient overdetermination factor is achieved. Note that at least one time shift is required in order to accurately estimate conjugate modal frequencies. This allows for the coefficients of the matrix coefficient, characteristic polynomial to be identified for a given model order m. The roots of this polynomial can be used to find the modal parameters.

The models represented by Equation (1) and Equation (3) are referred to as a **Unified Matrix Polynomial Approach (UMPA)**, models. Both models yield a matrix coefficient, characteristic polynomial (the $[\alpha]$ polynomial in these models). Equation (3) corresponds to a time domain AutoRegressive-Moving-Average (ARMA(m,n)) model, or more properly an AutoRegressive with eXogenous inputs (ARX(m,n)) model, that is developed from a set of discrete time equations. Since both the frequency and time domain models are based upon functionally similar matrix coefficient, characteristic polynomials, the UMPA(m,n) terminology will be used for models in both domains to reflect the order of the denominator polynomial (m) and the order of the numerator polynomial (n). In Section 2.2, this notation will be extended to UMPA(m,n,v) to reflect the order v of the base vector involved in the basic UMPA formulation.

In light of the above discussion, it is now apparent that most of the modal parameter estimation processes available could have been developed by starting from a general matrix polynomial formulation that is justifiable based upon the underlying matrix differential equation. The general matrix polynomial formulation yields essentially the same form of matrix coefficient, characteristic polynomial equation, for both time and frequency domain data.

For the frequency domain data case, this yields:

$$\left| [\alpha_m] s^m + [\alpha_{m-1}] s^{m-1} + [\alpha_{m-2}] s^{m-2} + \dots\dots\dots + [\alpha_0] \right| = 0 \tag{4}$$

where:

$$s_r = \lambda_r \qquad \lambda_r = \sigma_r + j\, \omega_r \tag{5}$$

For the time domain data case, this yields:

$$\left| [\alpha_{m}]\, z^{m} + [\alpha_{m-1}]\, z^{m-1} + [\alpha_{m-2}]\, z^{m-2} + \ldots\ldots\ldots + [\alpha_{0}] \right| = 0 \tag{6}$$

where:

$$z_r = e^{\lambda_r\, \Delta t} \qquad \lambda_r = \sigma_r + j\, \omega_r \tag{7}$$

$$\sigma_r = Re\left[\frac{\ln z_r}{\Delta t}\right] \qquad \omega_r = Im\left[\frac{\ln z_r}{\Delta t}\right] \tag{8}$$

Once the matrix coefficients ($[\alpha]$) have been found, the modal frequencies (λ_r or z_r) can be found as the roots of the matrix coefficient polynomial (Equation (4) or (6)) using any one of a number of numerical techniques, normally involving the companion matrix associated with the matrix coefficient polynomial.

When the modal frequencies are estimated from the eigenvalue-eigenvector problem that is associated with solving this matrix coefficient polynomial equation, a unique estimate of the unscaled modal vector is identified at the same time. The length or dimension of this unscaled modal vector is equal to the dimension of the square alpha coefficients which, in general, is equal to the row dimension of the FRF data matrix in order for the matrix coefficient polynomial equation to be conformal. Normally, this row dimension associated with the FRF or IRF data matrix is assumed to be connected with the number of outputs (N_o) that were measured.

Since the data matrix (FRF or IRF) is considered to be symmetric or reciprocal, the data matrix can be transposed, switching the effective meaning of the row and column index with respect to the physical inputs and outputs.

$$[\ H(\omega_i)\ ]_{N_o \times N_i} = [\ H(\omega_i)\ ]^T_{N_i \times N_o} \tag{9}$$

Since many modal parameter estimation algorithms are developed on the basis of either the number of inputs (N_i) or the number of outputs (N_o), assuming that one or the other is larger based upon test method, some nomenclature conventions are required for ease of further discussion. In terms of the modal parameter estimation algorithms and the ultimate matrix coefficient, characteristic polynomial equation, it is more important to recognize whether the algorithm develops the square matrix coefficient on the basis of the larger (N_L) of N_i or N_o or the smaller (N_S) of N_i or N_o. For this reason, the terminology of *long* (larger of N_i or N_o) dimension or *short* (smaller of N_i or N_o) dimension is easier to understand without confusion. Using this approach, PTD, RFP and PLSCF are all short dimension methods where the vector found as part of the solution for poles is very small while ERA and PFD are long dimension methods where the vector found as part of the solution for poles is of full length, based upon measurement locations.

To eliminate possible confusion, in recent explanations of modal parameter estimation algorithms, the nomenclature of the number of outputs (N_o) and number of inputs (N_i) has been replaced by the length of the long dimension of the data matrix (N_L) and the length of the short dimension (N_S) regardless of which dimension refers to the physical output or input. This means that the above reciprocity relationship can be restated as:

$$[\ H(\omega_i)\ ]_{N_L \times N_S} = [\ H(\omega_i)\ ]^T_{N_S \times N_L} \tag{10}$$

Note that the reciprocity relationships in Equation (9) and (10) are a function of the common degrees of freedom (DOFs) in the short and long dimensions. If there are no common DOFs, there are no reciprocity relationships. Nevertheless, the importance of Equation 3 and 4 comes from the idea that the dimensions of the FRF matrix can be transposed and this affects the size of the square alpha coefficients in the matrix coefficient polynomal equation.

Finally, once the modal frequencies and unscaled modal vectors are estimated via the eigenvalue-eigenvector problem, the residues (numerators) of the partial fraction model of the FRF data matrix are used to estimate the final, scaled modal vectors and modal scaling. Note that the unscaled modal vector found in the eigenvalue-eigenvector problem is available to be used as a weighting vector in the estimation of the residues and, therefore, the final scaled modal vectors and modal scaling. Also note that this weighting vector may be of length equal to the long or short dimension, depending on the modal parameter estimation algorithm being used.

$$[\ H(\omega_i)\ ]_{N_o \times N_i} = \sum_{r=1}^{N} \frac{[\ A_r\ ]_{N_o \times N_i}}{j\omega_i - \lambda_r} + \frac{[\ A_r^*\ ]_{N_o \times N_i}}{j\omega_i - \lambda_r^*} = \sum_{r=1}^{2N} \frac{[\ A_r\ ]_{N_o \times N_i}}{j\omega_i - \lambda_r} \tag{11}$$

This process means that most modern modal parameter estimation algorithms are implemented in a two stage procedure that has three steps as follows:

Stage 1, Step 1

- Load Measured Data into Over-Determined Linear Equation Form.
    - Utilize Matrix Coefficient Polynomial Based Model (Equation 1, 2 or 3).
    - Find Scalar or Matrix Coefficients ($[\alpha_k]$ and $[\beta_k]$).
    - Implement for Various Model Orders (Consistency/Stability Diagram).

Stage 1, Step 2

- Solve Matrix Coefficient Polynomial for Modal Frequencies (Equation 4 or 6).
    - Formulate Eigenvalue-Eigenvector Problem.
    - Eigenvalues Determine the Modal Frequencies (λ_r).
    - Eigenvectors Determine the Unscaled Modal Vectors ($\{\psi_r\}$) of dimension N_S or N_L.

Stage 2, Step 3

- Load Measured Data Into Over-Determined Linear Equation Form (Equation 11).
    - Determine Modal Vectors and Modal Scaling from Residues.

The most commonly used modal identification methods can be summarized as shown in Table 1. The high order model is typically used for those cases where the system is undersampled in the spatial domain. For example, the limiting case is when only one measurement is made on the structure. For this case, the left hand side of the general linear equation corresponds to a scalar polynomial equation with the order equal to or greater than the number of desired modal frequencies. The low order model is used for those cases where the spatial information is complete. In other words, the number of physical coordinates is greater than the number of desired modal frequencies. For this case, the order of the lefthand side of the general linear equation is equal to two. The zero order model corresponds to the case where the temporal information is neglected and only the spatial information is used. These methods directly estimate the eigenvectors as a first step. In general, these methods are programmed to process data at a single temporal condition or variable. In this case, the method is essentially equivalent to the single-degree-of-freedom (SDOF) methods which have been used with frequency response functions. In others words, the zeroth order matrix polynomial model compared to the higher order matrix polynomial models is similar to the comparison between the SDOF and MDOF methods used historically in modal parameter estimation.

	Domain		Matrix Polynomial Order			Coefficients	
Algorithm	Time	Freq	Zero	Low	High	Scalar	Matrix
Complex Exponential Algorithm (CEA)	•				•	•	
Least Squares Complex Exponential (LSCE)	•				•	•	
Polyreference Time Domain (PTD)	•				•		$N_S \times N_S$
Ibrahim Time Domain (ITD)	•			•			$N_L \times N_L$
Multi-Reference Ibrahim Time Domain (MRITD)	•			•			$N_L \times N_L$
Eigensystem Realization Algorithm (ERA)	•			•			$N_L \times N_L$
Polyreference Frequency Domain (PFD)		•		•			$N_L \times N_L$
Simultaneous Frequency Domain (SFD)		•		•			$N_L \times N_L$
Multi-Reference Frequency Domain (MRFD)		•		•			$N_L \times N_L$
Rational Fraction Polynomial (RFP)		•			•	•	$N_S \times N_S$
Orthogonal Polynomial (OP)		•			•	•	$N_S \times N_S$
Polyreference Least Squares Complex Frequency (PLSCF)		•			•	•	$N_S \times N_S$
Rational Fraction Polynomial-Z Domain (RFP-Z)		•			•	•	$N_S \times N_S$
Complex Mode Indication Function (CMIF)		•	•				$N_L \times N_S$

TABLE 1. Summary of Modal Parameter Estimation Algorithms

A number of extensions or clarifications of how different modal parameter estimation algorithms fit into the UMPA (m,n) model are useful when an autonomous modal parameter estimation procedure is developed. These considerations are covered in the following sections.

2.2 Base Vector Order

When formulating the basic UMPA(m,n) equation for various model orders, the resulting matrix coefficient polynomial involves coefficient matricies which are sized based upon the short or long dimension of the data matrix. Once this matrix coefficient polynomial is chosen, the set of unscaled modal vectors, or base vectors is effectively chosen where the length of each vector matches this dimension. For model orders equal to one, however, this matrix model will not be able to estimate complex conjugate solutions without at least one time shift (time domain implementation) or one derivative (frequency domain implementation) when the model is formulated. This means that the base vector is a state vector for this system. Instead of the base vector being of zeroth order as for all other UMPA cases, the base vector will be of order one. If successive time shifts or derivatives are included, the base vector will be of higher order. The form of the base vector is as follows where v is the order of the base vector:

$$\{\phi\}_r = \begin{Bmatrix} \lambda_r^v \{\psi\}_r \\ \cdot \\ \cdot \\ \cdot \\ \lambda_r^2 \{\psi\}_r \\ \lambda_r^1 \{\psi\}_r \\ \lambda_r^0 \{\psi\}_r \end{Bmatrix}_r \tag{12}$$

This means that the notation for the UMPA(m,n) model does not completely define the UMPA formulation. A more correct formulation would be UMPA(m,n,v). For most commercial algorithms, v is normally zero and the base vector is zeroth order. For the Eigensystem Realization Algorithm (ERA) and the first order version of the Polyreference Frequency Domain (PFD) algorithm, v is one and the base vector is first order. While no commercial algorithm utilizes v greater than one, there is no reason to restrict the UMPA(m,n,v) formulation to zeroth and first order base vectors. Allowing the base vector to take on higher orders lengthens the vector that will be evaluated for correlation or consistency among all of the possible solutions. This is an extremely useful concept when developing autonomous modal parameter estimation procedures.

2.3 Equation Normalization

In formulating the least squares estimate of the coefficient matrices of the matrix coefficient polynomial, a decision must be made on how to normalize the matrix coefficient polynomial so that one of the coefficient matrices is an identity matrix. Essentially, for an m-th order, matrix coefficient polynomial, this means that one of the two following forms must be chosen:

$$\left| \lceil I \rfloor s^m + [\alpha_{m-1}]\, s^{m-1} + [\alpha_{m-2}]\, s^{m-2} + \ldots\ldots\ldots + [\alpha_0] \right| = 0 \tag{13}$$

$$\left| [\alpha_m]\, s^m + [\alpha_{m-1}]\, s^{m-1} + [\alpha_{m-2}]\, s^{m-2} + \ldots\ldots\ldots + \lceil I \rfloor \right| = 0 \tag{14}$$

From a theoretical viewpoint, these two equations appear to be the same equation as long as the coefficient matrices are invertible. Numerically, though, this choice of normalization changes how the noise is minimized in the least squares procedure and is similar to the H_1 and H_2 estimation differences in frequency response function estimation. This concept has been studied in some detail and has become the central issue in generating consistency diagrams that are much easier to understand (clear stabilization or consistency diagrams) [47,52-53]. When developing autonomous modal parameter estimation procedures, allowing both normalizations increases the number of solutions that will be compared when estimating vector correlation or consistency. This is also an extremely useful concept when developing autonomous modal parameter estimation procedures.

3. Background: Autonomous Modal Parameter Estimation

The interest in automatic modal parameter estimation methods has been documented in the literature since at least the mid 1960s when the primary modal method was the analog, force appropriation method [1-3]. Following that early work, there has been a continuing interest in autonomous methods [9-28] that, in most cases, have been procedures that are formulated based upon a specific modal parameter estimation algorithm like the Eigensystem Realization Algorithm (ERA), the Polyreference Time Domain (PTD) algorithm or more recently the Polyreference Least Squares Complex Frequency (PLSCF) algorithm or the commercial version of the PLSCF, the PolyMAX ® method.

Each of these past procedures have shown some promise but have not yet been widely adopted. In many cases, the procedure focussed on a single modal parameter estimation algorithm and did not develop a general procedure. Most of the past procedural methods focussed on pole density but depended on limited modal vector data to identify correlated solutions. Currently, due to increased computational speed and larger availability of memory, procedural methods can be developed that were beyond the computational scope of available hardware only a few years ago. These methods do not require any initial thresholding of the solution sets and rely upon correlation of the vector space of thousands of potential solutions as the primary identification tool. With the addition to any modal parameter estimation algorithm of the concept of pole weighted base vector, the length, and therefore sensitivity, of the extended vectors provides an additional tool that appears to be very useful.

4. Numerical Tools - Autonomous Modal Parameter Estimation

The development of many of the past procedures and any new proposed procedure depends heavily on a number of numerical tools that have been developed over the last twenty years or so [29-54]. These tools are currently used by many algorithms as a user interaction tool or semi-autonomous tool to assist the user in picking an appropriate set of modal parameters. These tools are described in great detail in the literature and are summarized briefly in the next several sections.

4.1 Consistency Diagrams

For the last thirty years, modal parameter estimation based upon experimental data, primarily frequency response functions (FRFs), has utilized some form of error chart and/or stabilization diagram to visualize and assist in the determination of the correct modal frequencies [46-49,52-53]. The conceptual basis of the stabilization diagram is that distinct and unique modal frequencies can be identified by comparing the roots of a characteristic polynomial when the model order of the characteristic polynomial is increased or the subspace is altered in a systematic manner. The stabilization or consistency diagram is based upon successive solutions of Equation (4) or Equation (6) for different values of the maximum model order m. If the roots are consistent as the model order is increased, these roots are identified as modal frequencies. If the roots are inconsistent, these roots are associated with noise on the data and are discarded. Since there is generally much more FRF data available than is needed to solve for the number of modal parameters of interest, the characteristic polynomial is normally estimated in a least squares sense and can be reformulated from the measured data for each model order.

In most implementations, the **stabilization diagram** or **consistency diagram** is presented as a two dimensional plot with frequency on the abscissa and characteristic polynomial model order on the ordinate. A plot of the summed magnitude of all of the FRFs, or a plot of one of the mode indicator functions (complex mode indicator function (CMIF) or multivariate mode indicator function (MvMIF)), is placed in the background for reference. For each model order, symbols are plotted along the frequency axis wherever a root of the characteristic polynomial has been estimated. Historically, the characteristic polynomial had scalar coefficients (single reference data) so the roots are complex-valued and have both frequency and damping information. The original symbols were used to indicate, in increasing importance, Level 1: a pole was found, Level 2: a pole was found with an associated complex conjugate, Level 3: Level 2 plus the damping was realistic (negative real part of pole), Level 4: Level 3 plus a damped natural frequency (imaginary part of pole) consistency within a specified percentage (normally 1 percent) and Level 5: Level 4 plus a damping consistency with a specified percentage (normally 5 percent).

As multiple reference parameter estimation was developed, this led to matrix coefficient, characteristic polynomial equations which created further evaluation criteria, Level 6: Level 5 plus a vector consistency within a specified modal assurance criterion (MAC) value (normally 0.95). Since the vector that is associated with the matrix coefficient, characteristic polynomial may be different sizes (function of N_S or N_L), a variation of Level 6 involves an additional solution for the vector at the largest dimension N_L so that a more statistically significant comparison of vectors can be utilized. Finally, numerical conditioning can be evaluated for each solution and if the numerical conditioning is approaching a limit based upon the accuracy of the data or the numerical limitations of the computer algorithm or word size, Level 7 can be added to indicate a possible numerical problem. In general, all of the specified values that indicate consistency at each level can be user defined and are referred to as the stability or consistency tolerances.

This information generates a consistency diagram, for a simple circular plate structure, that looks like Figure 1. With all of the symbols presented, the consistency diagram for even this simple structure can be very complicated.

Figures 1 and 2 demonstrate two different presentations of consistency diagrams based upon different presentations of the characteristic matrix coefficient polynomial information. In both figures, an average autopower of the measured data is plotted on the consistency diagram in the background for reference.

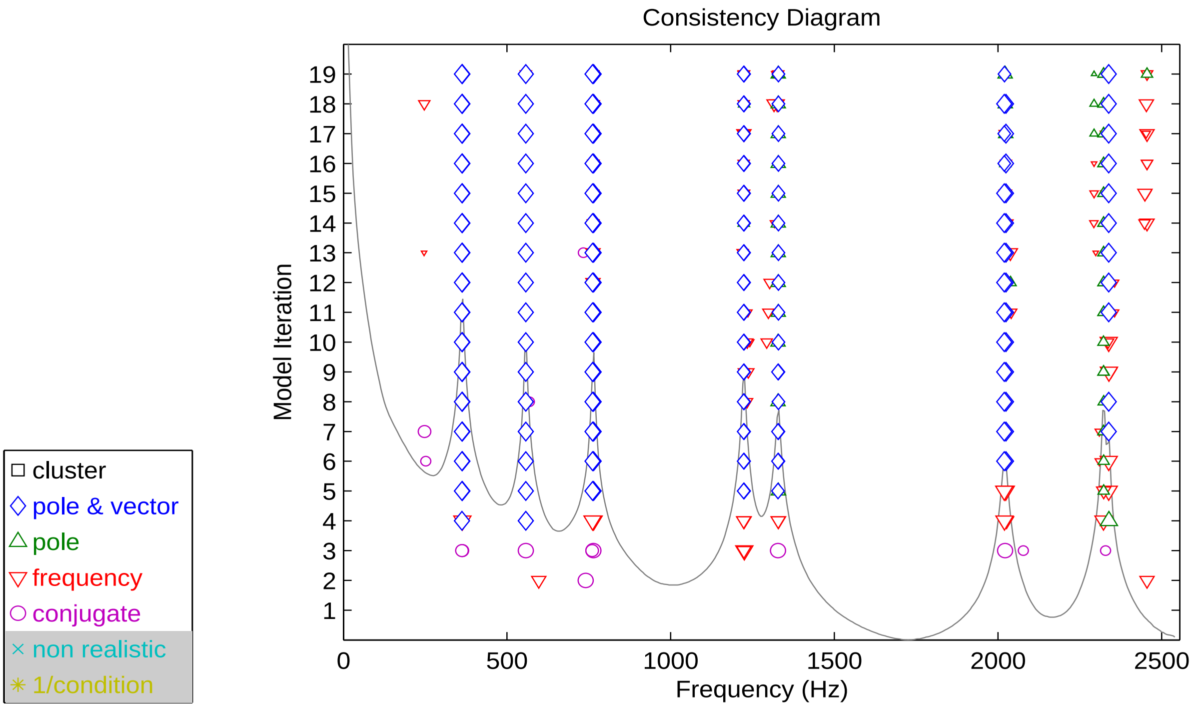

Figure 1. Typical Consistency Diagram

Figure 2 presents the consistency diagram using an entirely different method that involves using the modal assurance criterion (MAC) to compare vectors from successive solutions of the characteristic polynomial equation. Rather than comparing the unscaled modal vectors (base vectors) directly, a pole weighted vector of v-th order is constructed for each solution and compared to similarly constructed pole weighted vectors for the previous solution. Further explanation of the pole weighted vector is provided in Section 4.3 and can also be found in the literature [46].

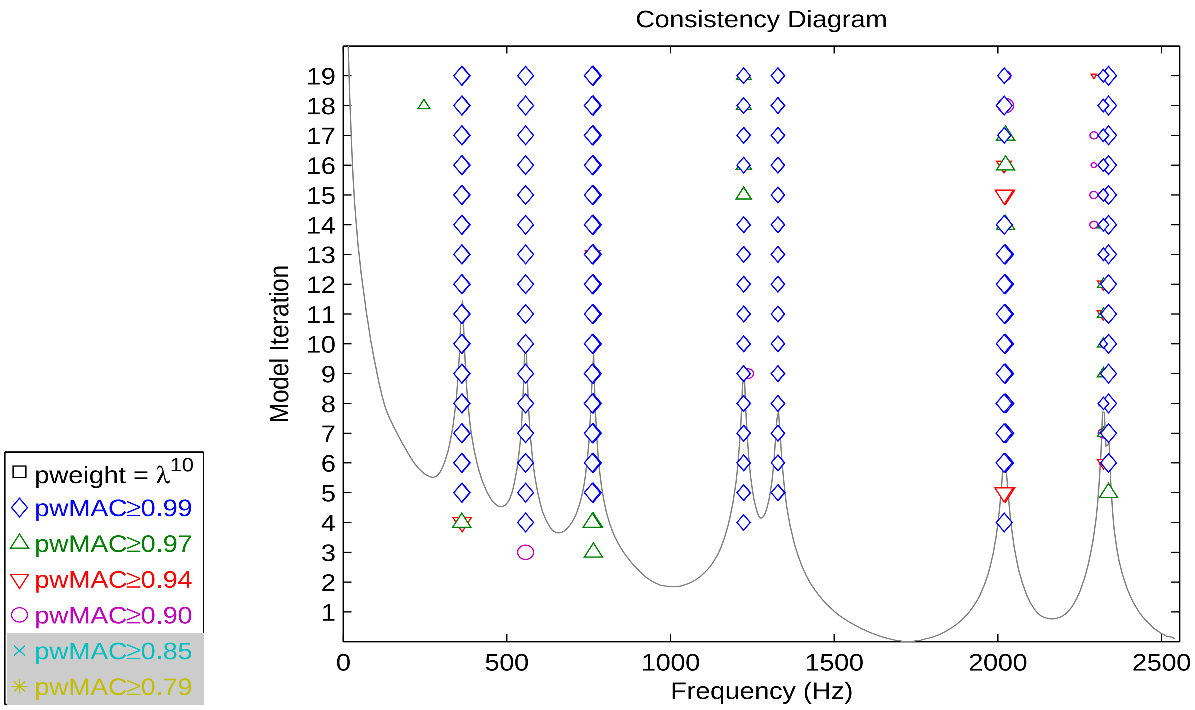

Figure 2. Alternate Consistency Diagram - Pole Weighted MAC

4.1.1 Clear Consistency Diagrams

A number of different methods can be used to generate the consistency diagram that will impact the clarity of the consistency diagram. A number of recent papers [46-47,52-53] have identified the effects of changing the consistency tolerances on the resultant consistency diagrams, yielding a clearer presentation of symbols that indicate the presence of a structural mode of vibration. These methods can be combined with the coefficient normalization and consistency tolerances to generate a very clear diagram in most cases where the measured data has a reasonable match to linear, reciprocal system assumptions, observability issues and reasonable data noise levels. These methods have been used on a wide range of data cases in the automotive and aerospace application areas with good success. The methods include:

- Symbol Sizing Based Upon Normal Mode Criteria
- Complete or Incomplete Vector Comparisons
- Using Both Coefficient Normalization Methods
- Numerator and Denominator Model Order Variation
- Fixed Denominator and Numerator Order Variation
- Frequency Normalization Variation

4.2 Long Dimension Vector Solution

While many high order, matrix coefficient modal parameter estimation methods estimate unscaled modal vectors as part of the estimation of the poles, there is no reason to limit the length of the unscaled modal vector to the short dimension. Each short dimension vector can be used to estimate the unscaled vector for the long dimension as a part of the solution. This requires an extra solution step but, for each model order, requires very little additional computational effort. In this case, regardless of the method employed to estimate the modal frequencies, all unscaled modal vectors will be of length equal to the long dimension. No attempt to restrict the set of modal frequencies is used at this point in the procedure; all possible poles are included in this calculation. This set of unscaled vectors will include structural modal vectors and computational vectors. Sorting these vectors is left to a correlation procedure such as the modal assurance criterion (MAC) with a threshold (minimum MAC value).

4.3 Pole Weighted Modal Vectors

When comparing base vectors, at either the short or the long dimension, a pole weighted base vector can be constructed independent of the original UMPA(m,n,v) procedure used to estimate the poles and base vectors. For a given order v of the pole weighted vector, the base vector and the associated pole can be used to formulate the pole weighted vector as follows:

$$\{\phi\}_r = \begin{Bmatrix} \lambda_r^v \{\psi\}_r \\ \cdot \\ \cdot \\ \cdot \\ \lambda_r^2 \{\psi\}_r \\ \lambda_r^1 \{\psi\}_r \\ \lambda_r^0 \{\psi\}_r \end{Bmatrix}_r \tag{15}$$

The above formulation will be dominated by the high order terms if actual frequency units are utilized. Generalized frequency concepts (frequency normalization or z domain transform) are normally used to minimize this problem as follows:

$$\{\phi\}_r = \begin{Bmatrix} z_r^v \{\psi\}_r \\ \cdot \\ \cdot \\ \cdot \\ z_r^2 \{\psi\}_r \\ z_r^1 \{\psi\}_r \\ z_r^0 \{\psi\}_r \end{Bmatrix}_r \tag{16}$$

$$z_r = e^{j*\pi*(\omega_r/\omega_{max})} = e^{j*\omega_r*\Delta t} \tag{17}$$

$$z_r^m = e^{j*\pi*m*(\omega_r/\omega_{max})} \tag{18}$$

In the above equations, Δt and ω_{max} can be chosen as needed to cause the positive and negative roots to wrap around the unit circle in the z domain without overlapping (aliasing). Normally, ω_{max} is taken to be five percent larger than the largest frequency identified in the roots of the matrix coefficient polynomial.

4.4 Pole Surface Consistency and Density

A number of other pole presentation diagrams, related to the consistency diagram, such as pole surface consistency and pole surface density diagrams have proven useful for identifying modal parameters [11-13] and may be more powerful than the consistency diagram alone. Generally, pole surface density diagrams are more powerful than consistency or stability diagrams at locating similar pole vector estimates from all of the possible solutions represented in the consistency diagram. All of the poles from all solutions involved in the consistency diagram are ploted in the second quadrant of the s plane. Pole estimates that are located within a two dimensional theshold from each other are defined as participating in a pole cluster or a dense region of estimated poles in the s plane. The poles that are compared on these pole surface diagrams are generally limited to the poles identified on the consistency diagram (if some symbols are omitted from the consistency diagram, these poles will not be included on the pole surface diagram. The distribution of the poles that participate in a pole cluster can be used to find a single pole-vector estimate and the distribution can be used to estimate statistics related to the variance in the pole estimate. An example of a pole surface consistency diagram is given in Figure 3 and the companion pole surface density diagram in Figure 4.

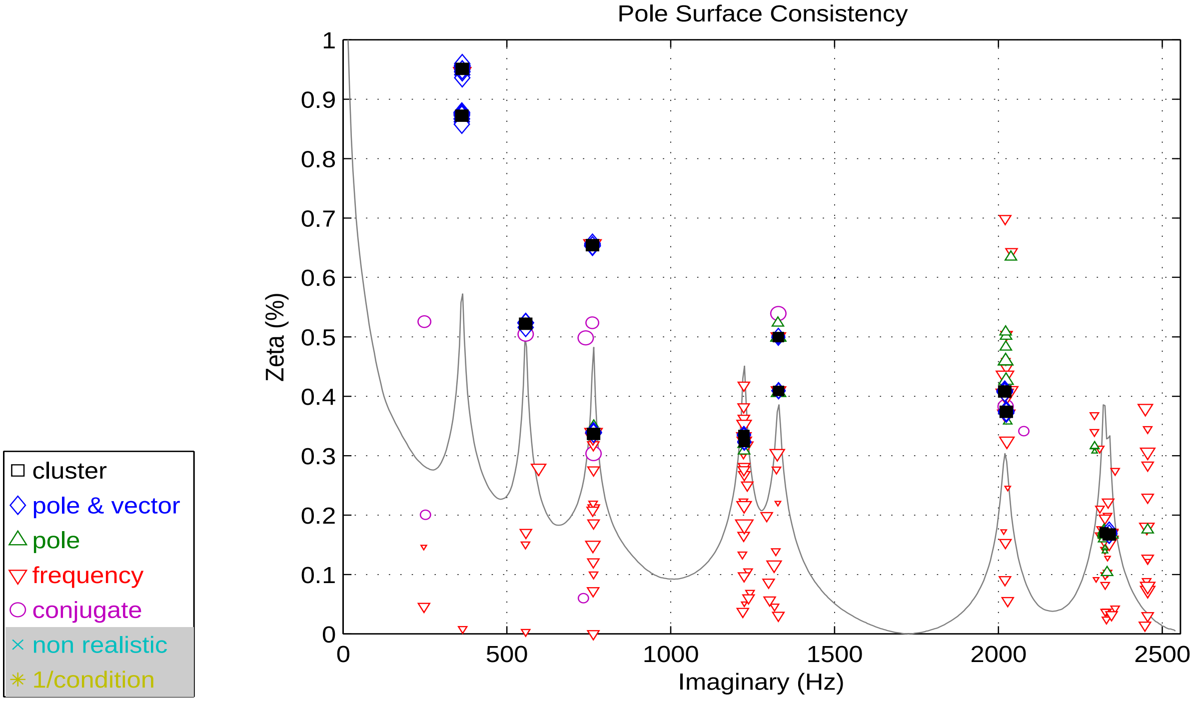

Figure 3. Pole Surface Consistency Diagram with Final Autonomous Estimates

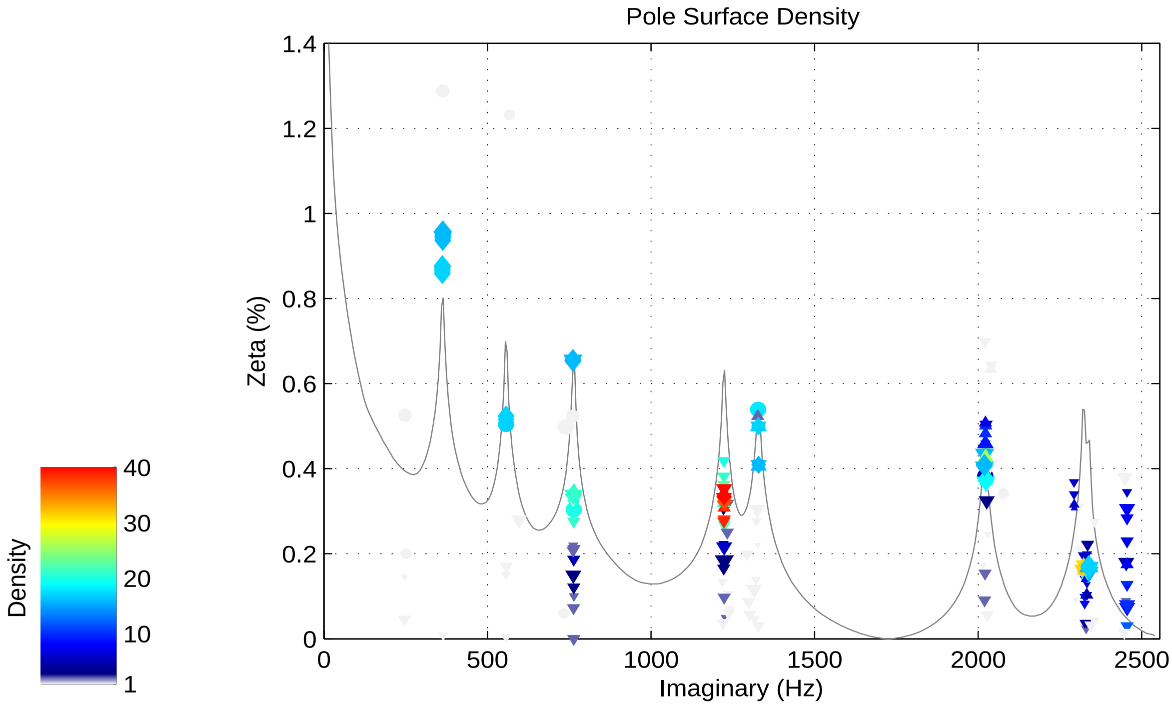

Figure 4. Pole Surface Density Diagram

Figures 3 and 4 presents the same information previously shown in Figures 1 and 2. Note that in all cases, suppressing the obvious spurious computational poles results in a substantially cleaner consistency diagram with clear indications of the pole locations. This translates to the Pole Surface Diagram as well.

5. Autonomous Modal Parameter Estimation Methods

Two methods are presented that both yield good results for a variety of realistic applications with little or no user modification of thresholds or other parameters that are used to control the autonomous method.

5.1 Autonomous First Order Model, Fifth Order Base Vector

The first autonomous method presented is based upon a low order, time domain algorithm similar to the Eigensystem Realization Algorithm (ERA). In the nomenclature used in this paper, the ERA method is an UMPA(1,0,1) method meaning that it is a first order matrix coefficient polynomial with a zeroth order numerator polynomial and a first order base vector. The method used for the autonomous method is an UMPA(1,0,5) meaning that it utilizes a fifth order base vector found by using more starting times than the ERA method. By creating this new algorithm, the dimension of the base vector is much larger allowing a vector comparison between two equation normalization approaches to define the set of modal parameters. Complete details concerning this extended state concept may be found in the literature [31]. While the method will be demonstrated for a time domain solution, either time or frequency domain solutions that are UMPA(1,0,5) methods will work similarly. While a fifth order base vector is used in this case, the choice of any reasonable order seems to work equally well.

The implementation of the autonomous modal parameter estimation for this method is as follows:

- Solve for poles and extended base vectors using lead coefficient normalization ($[\alpha_1] = [\mathrm{I}]$).
- Solve for poles and extended base vectors using lead coefficient normalization ($[\alpha_0] = [\mathrm{I}]$).

- Sort each set of solutions into frequency order based upon damped natural frequency (ω_r) order.
- Compute the Cross MAC matrix between these two sets of base vector solutions.
- Retain only those vectors that have a Cross MAC above a threshold, typically 0.99.
- For matched pairs of base vectors satisfying this criteria, find the average pole (λ_r) and the average modal vector ($\{\psi_r\}$). This represents the final set of modal frequencies, modal damping values and mode shapes.
- For the modal parameters identified, complete the solution for modal scaling using any MIMO process of your choosing. A complete discussion of the choices can be found in the literature [48].

Once the final set of modal parameters is obtained, quality can be assessed by performing comparisons between the original measurements and measurements synthesized from the modal parameters. Other evaluations that may be helpful can be mean phase correlation (MPC) on the vectors, an Auto MAC looking for agreement between the modal vectors from conjugate poles or any other method available.

5.1.1 Application Example: H-Frame Laboratory Test Data

As an example of this autonomous modal parameter estimation method, sample data from a laboratory test structure known as an H-Frame is processed. This structure is relatively simple but provides some difficulty for modal parameter estimation due to the torsion modes involving each leg of the H-Frame as well as a number of nearly identical repeated roots. The FRF data in this case has 231 inputs and 25 responses. The first order, time domain model with a fifth order base vector successfully processed this data with little difficulty. In this case, the default threshold of 0.99 between the two sets of estimates was used.

For this test example, Figures 5-10 represent information used to evaluate the success of the autonomous procedure. These figures are used after the modal parameters have been estimated to assess reasonableness and are not used to guide the procedure. Obviously, when the procedure is complete if modes have been missed or misidentified, adjustments in the control parameters (base vector order and cross MAC threshold) can be made and the procedure repeated. An experienced user may wish to add or delete modes in a manual interaction as is currently done.

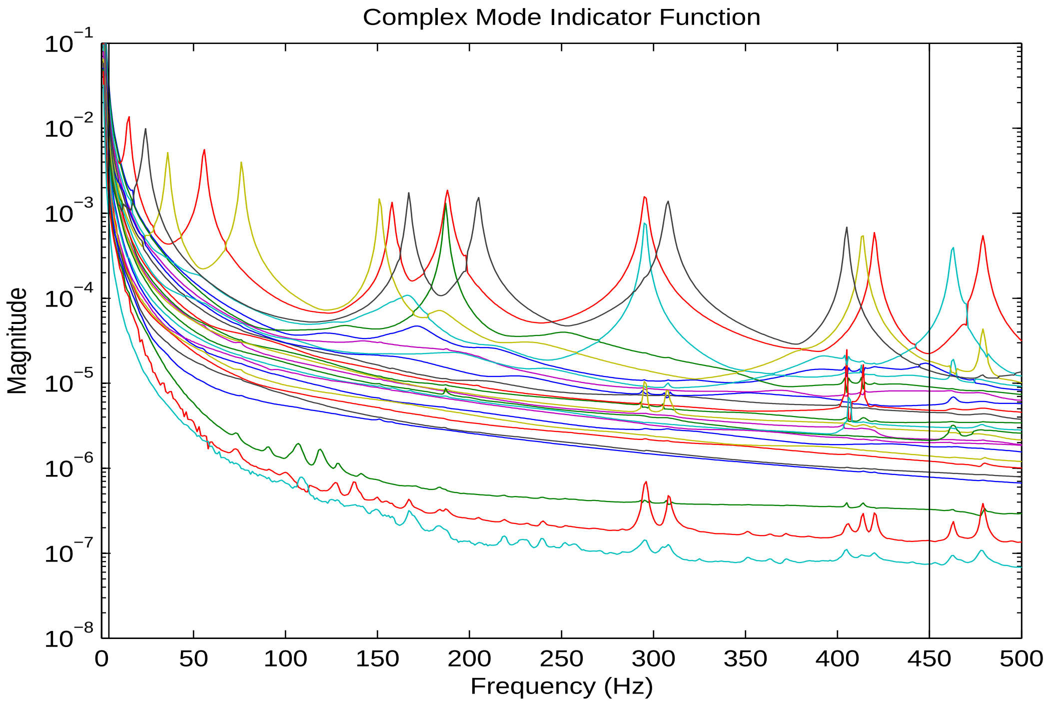

Figure 5. Complex Mode Indicator Function (CMIF)

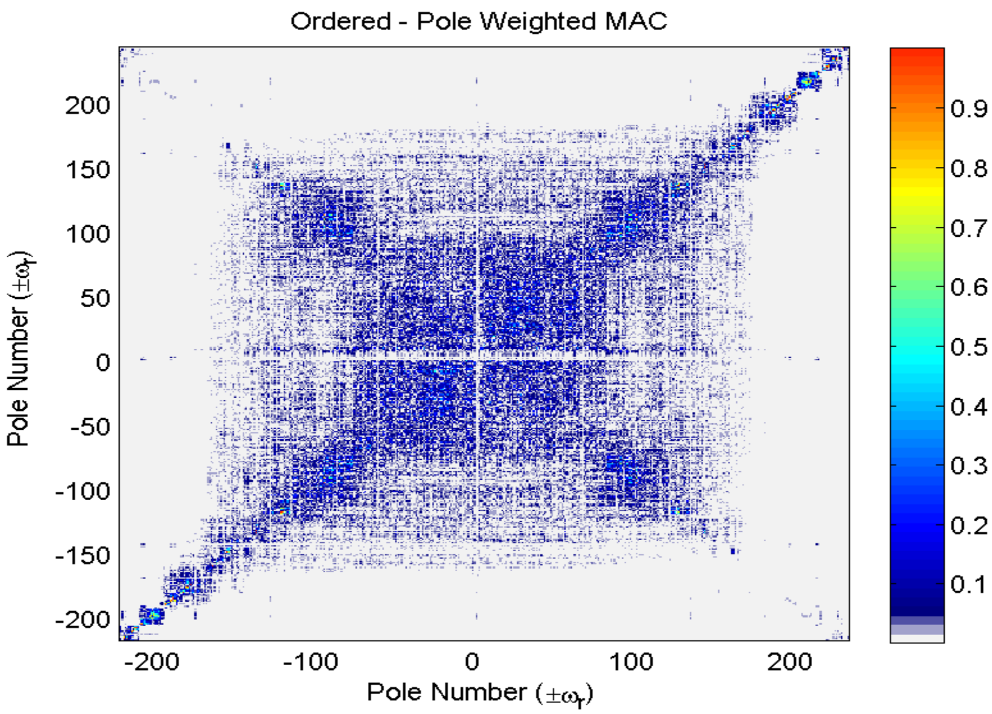

Figure 6. MAC-Fifth Order Pole Weighted Vectors, No Threshold

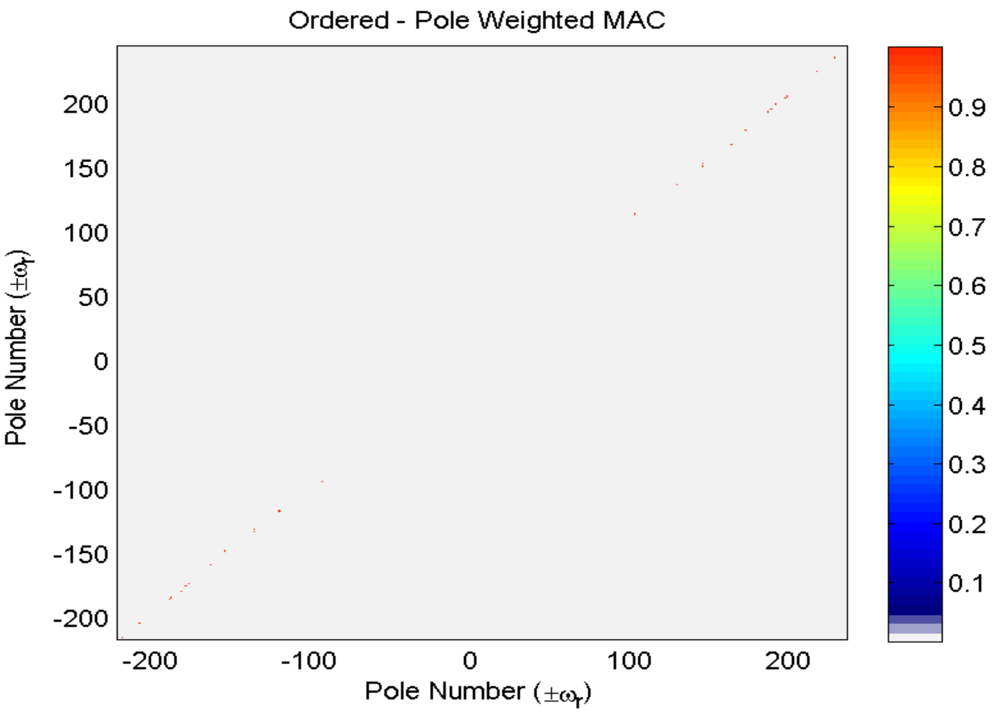

Figure 7. MAC-Fifth Order Pole Weighted Vectors, Above Threshold

Note that there are identified pole weighted vectors in Figure 7 along the diagonal but they are individual points and are hard to see in this plot. The interested reader will need to expand the graphical plot in the PDF version of this paper in order to see the solutions.

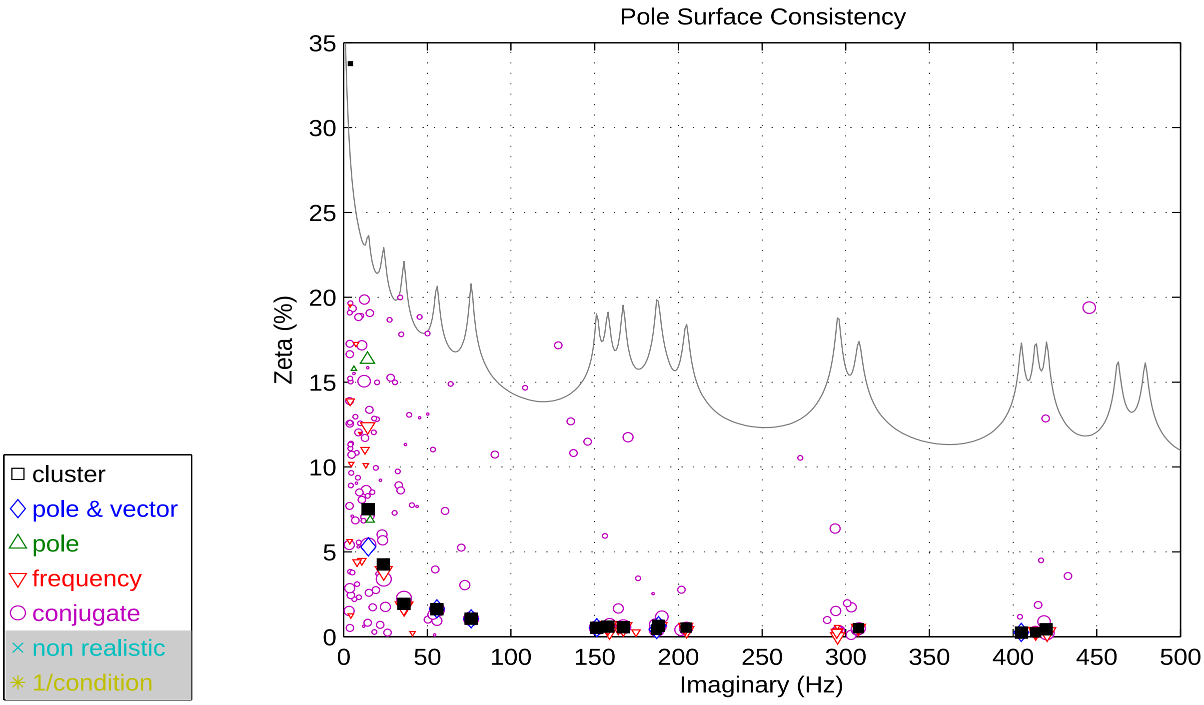

Figure 8. Pole Surface Consistency Clusters with Final Autonomous Estimates

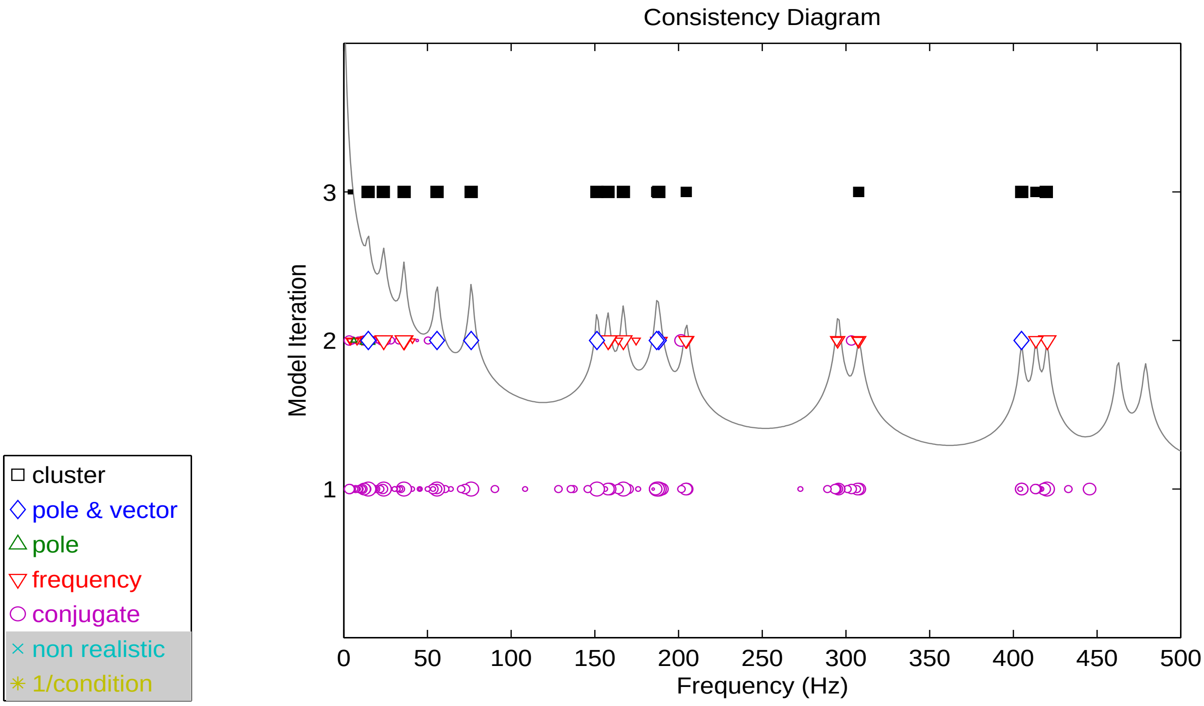

Figure 9. Consistency Diagram with Final Autonomous Estimates

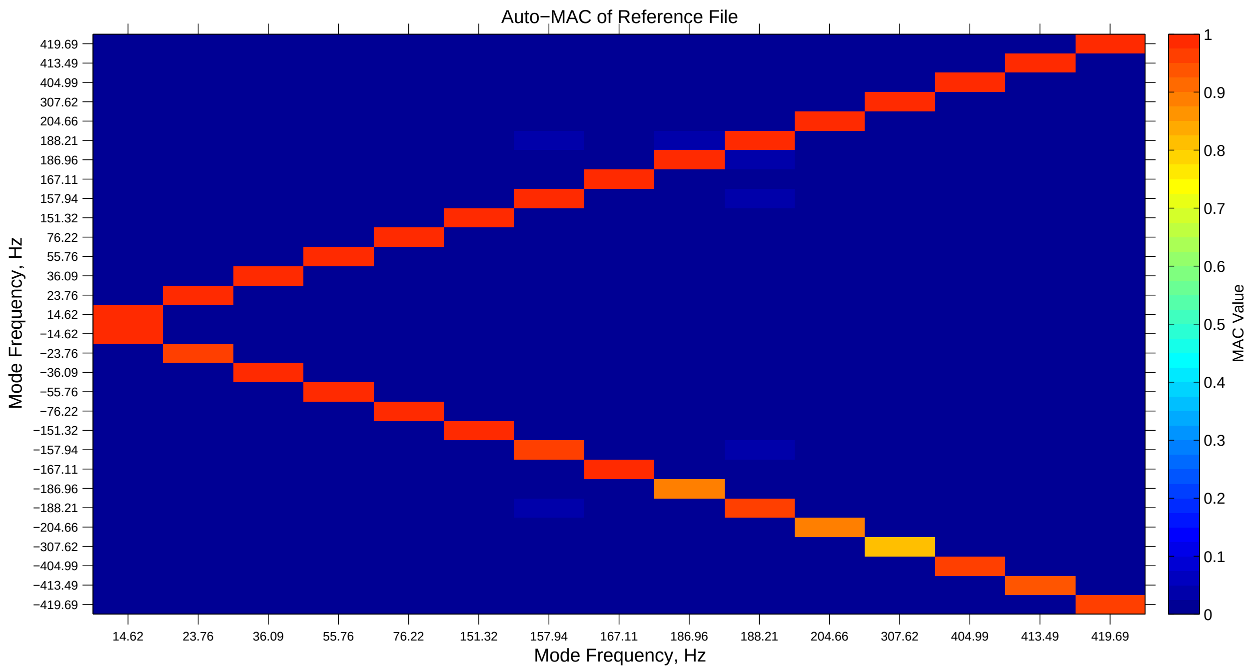

Figure 10. Auto MAC of Final Autonomous Estimates - Conjugate Poles

Figure 10 has been found to be a useful indicator when estimating modal parameters for a dominantly normal mode system. While structural dynamics theory requires that modal parmeters occur in conjugate pairs, there is no constraint in the solution method that requires the solution to have conjugate sets of modal parameters. The Auto MAC plot of the estimated modal vectors shown in Figure 10 shows a nearly perfect correlation between the vectors associated with the positive frequency and the vector associated with the estimate of its negative frequency (conjugate pair). In difficult modal parameter situations, this result has been found to be a strong indicator that the estimated modal model is reasonable.

5.2 Autonomous UMPA Method

The second autonomous method developed and presented in the following is a general method that can be used with any algorithm that fits within the UMPA structure. This means that this method can be applied to both low and high order methods with low or high order base vectors. This also means that most commercial algorithms could take advantage of this procedure. Note that high order matrix coefficient polynomials normally have coefficient matrices of dimension that is based upon the short dimension of the data matrix ($N_S \times N_S$). In these cases, it may be useful to use the extended base vector approach used in the previous autonomous method. This will extend the temporal-spatial information in the base vector so that the vector will be more sensitive to change. This characteristic is what gives this autonomous method the ability to distinguish between computational and structural modal parameters.

The implementation of the autonomous modal parameter estimation for this method is as follows:

- Develop a consistency diagram using any UMPA solution method. Since this autonomous method utilizes a pole surface density plot, having a large number of iterations in the consistency diagram (due to model order, subspace iteration, starting times, equation normalization, etc.) will be potentially advantageous. However, the larger the number of solutions (represented by symbols) in the consistency diagram, the more computation time and memory will be required. However, restricting the number of solutions using clear stabilization (consistency) methods may be counterproductive.
- If the UMPA method is high order (coefficient matrices of size $N_S \times N_S$), solve for the complete vector (function of N_L for all roots, structural and computational).
- Based upon the pole surface density threshold, identify all possible pole densities above some minimum value. This will be a function of the number of possible solutions represented by the consistency diagram.
- Sort the remaining solutions into frequency order based upon damped natural frequency (ω_r).
- Construct the 10th order, pole weighted vector for each solution.
- Normalize all pole weighted vectors to unity length with dominant real part.
- Calculate the Auto MAC matrix for all pole weighted vectors.
- Retain all Auto MAC values that have a pole weighted MAC value above a threshold, 0.8 works well for most cases. All values below the threshold are set to 0.0.
- Identify vector clusters from this pole weighted MAC diagram that represent the same pole weighted vector. This is done by a singular value decomposition (SVD) of the pole weighted MAC matrix. The number of significant singular values for this MAC matrix represents the number of significant pole clusters in the pole weighted vector matrix and the value of each significant singular value represents the size of the cluster since the vectors are unitary. Note that the singular value is nominally the number of vectors in the cluster and will likely be different, mode by mode.
- For each significant singular value, the location of the corresponding pole weighted vectors in the pole weighted vector matrix (index) is found from the associated left singular vector. This is accomplished by multiplying the left singular vector by the square root of the singular value and retaining all positions (indexes) above a threshold (typically 0.9). The positions of the non-zero elements in this vector are the indexes into the pole weighted vector matrix for all vectors belonging to a single cluster.
- For each identified pole cluster, perform a singular value decomposition (SVD) on the set of pole weighted vectors. The significant left singular vector is the dominant (average) pole weighted vector. Use the zeroth order portion of this dominant vector to identify the modal vector and the

relationship between the zeroth order and the first order portions of the dominant vector to identify the modal frequency and modal damping values.

- For the modal parameters identified, complete the solution for modal scaling using any MIMO process of your choosing. A complete discussion of the choices can be found in the literature [48]. A threshold can be used on the final modal scaling value to exclude all solutions with modal scaling below one percent of the maximum modal scaling found. (This has not been used on the data presented in the following sections.)

Once the final set of modal parameters is obtained, quality can be assessed by performing comparisons between the original measurements and measurements synthesized from the modal parameters. Other evaluations that may be helpful can be mean phase correlation (MPC) on the vectors, an Auto MAC looking for agreement between the modal vectors from conjugate poles or any other method available.

5.2.1 Application Example: C-Plate Laboratory Test Data

The first example of the autonomous UMPA method is performed on a laboratory test object consisting of a circular plate. This test object is very lightly damped and nearly every peak in the data is associated with a repeated root caused by the symmetry of the test object. This test object has been tested many times and the autonomous UMPA method estimated modal parameters consistent with past analysis. The FRF data in this case has 7 responses and 36 inputs (taken with an impact test method). In this case, a Polyreference Time Domain (PTD) method is used but, for every possible pole estimated over a model order range from 2 to 20, a complete long dimension modal vector was estimated so that sufficient spatial information (base vector of length 36) is available for sorting out the consistent solutions. For this case, the following thresholds and control parameters were used:

- Lowest order coefficient matrix normalization.
- Pole density threshold (4 and above).
- Pole weighted vector of model order 10.
- Pole weighted MAC threshold (0.8 and above).
- Cluster size threshold (4 and above).
- Cluster identification threshold (0.8 and above).

Figures 11-17 represent information used to evaluate the success of the autonomous procedure. These figures are used after the modal parameters have been estimated to assess reasonableness and are not used to guide the procedure. Obviously, when the procedure is complete if modes have been missed or misidentified, adjustments in the control parameters (base vector order and cross MAC threshold) can be made and the procedure repeated. An experienced user may wish to add or delete modes in a manual interaction as is currently done.

Figure 11 is the complex mode indicator function plot which is used to distinguish close or repeated modes. Figure 12 and 13 show the solutions that are remaining after the initial pole surface density threshold and pole weighted vector correlation threshold have been applied.

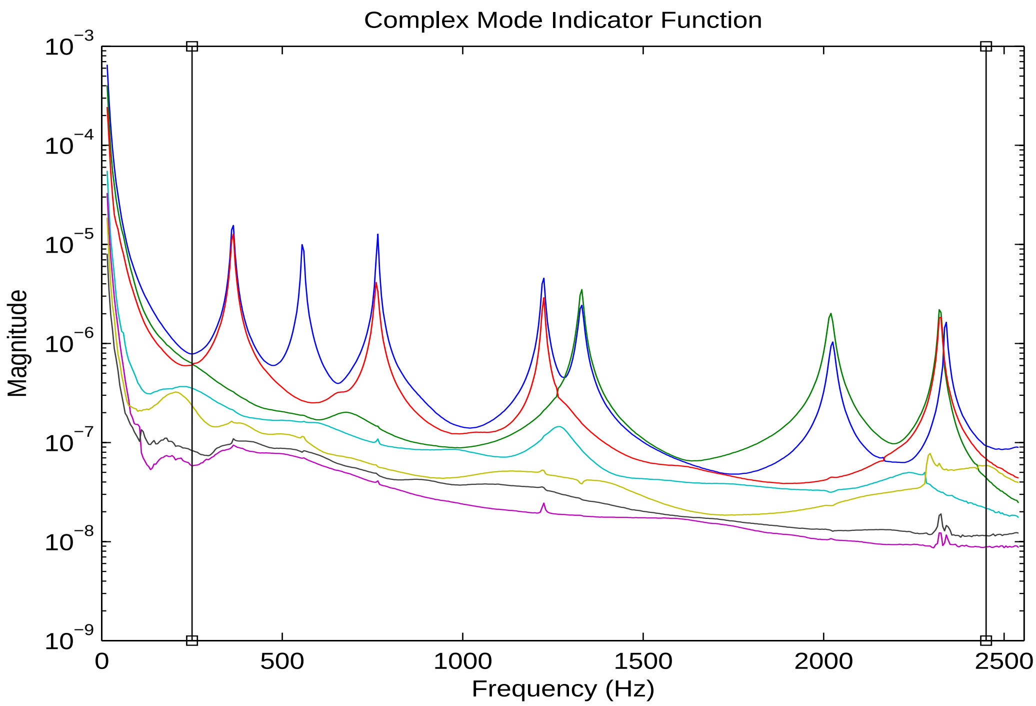

Figure 11. Complex Mode Indicator Function (CMIF)

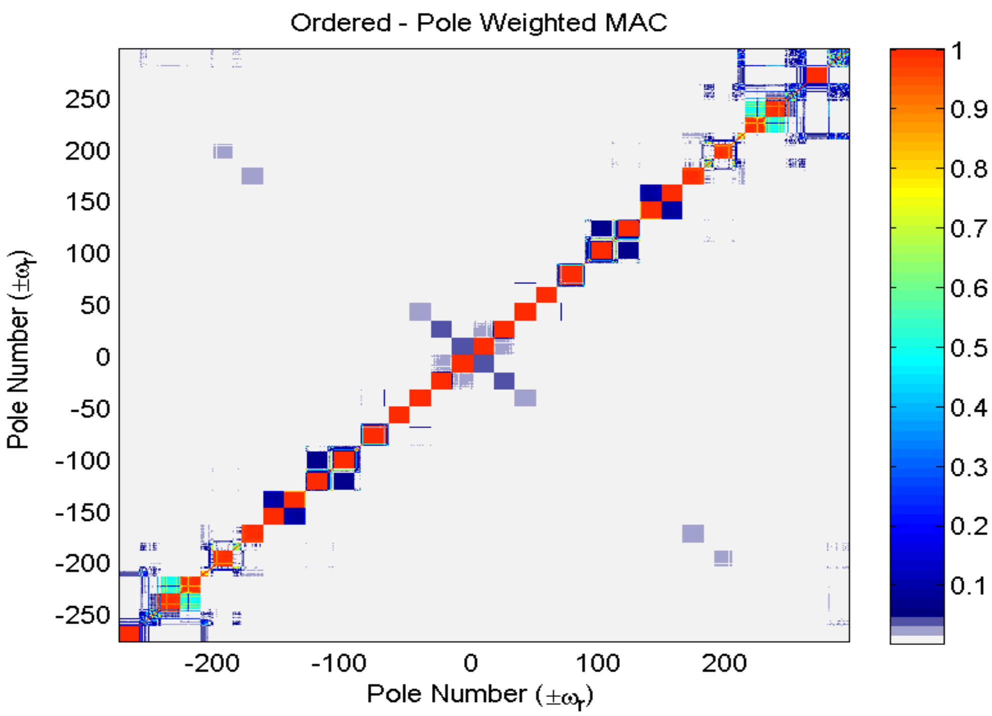

Figure 12. MAC-Tenth Order Pole Weighted Vectors, No Threshold

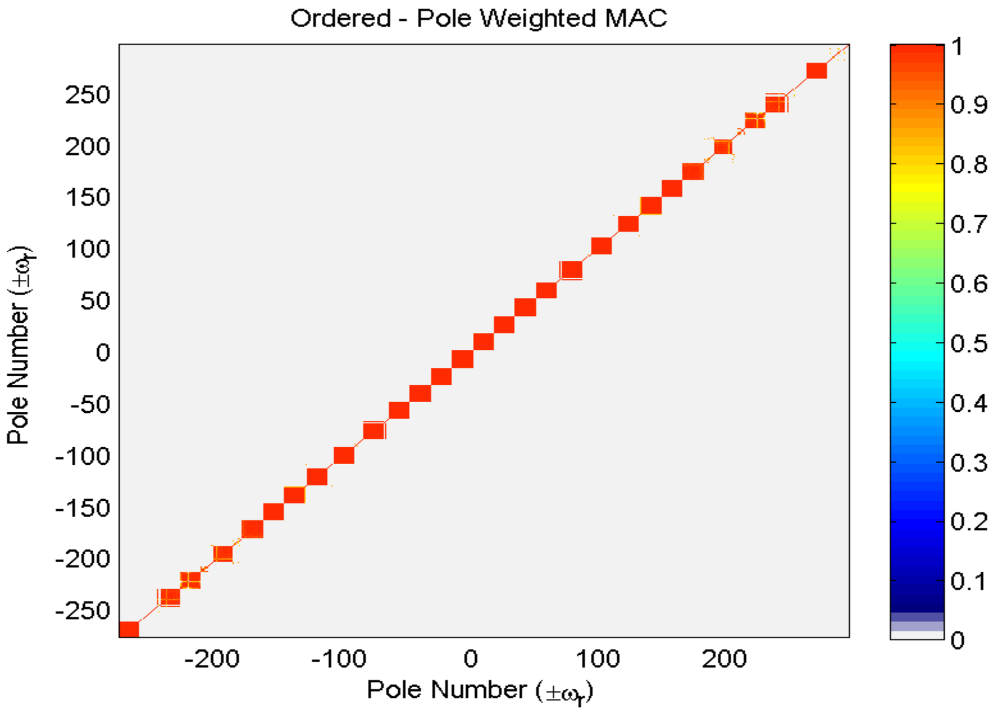

Figure 13. MAC-Tenth Order Pole Weighted Vectors, Above Threshold

The size of the red squares in Figure 13 represents the number of vectors in each cluster of poles found anywhere in the consistency diagram. Figure 14 is a plot of the scaled significant singular values for Figure 13. Only the singular values above the cluster size threshold are retained for the final solution. Figure 15 shows the location of the final estimates on a pole surface consistency plot. The black squares represent a final solution from the autonomous procedure.

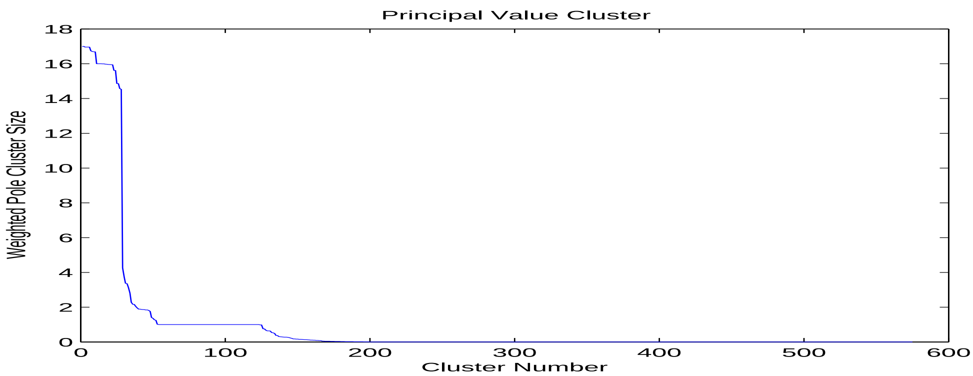

Figure 14. Principal Values of Clusters of Pole Weighted Vectors

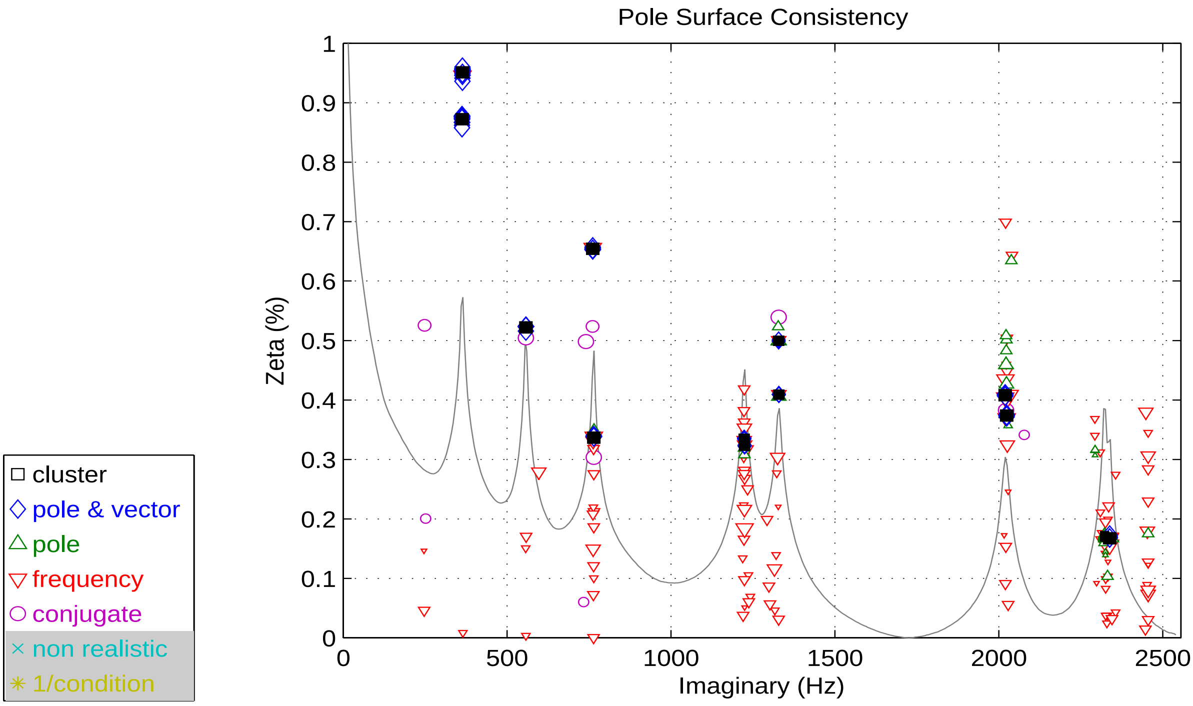

Figure 15. Pole Surface Consistency Clusters with Final Estimates

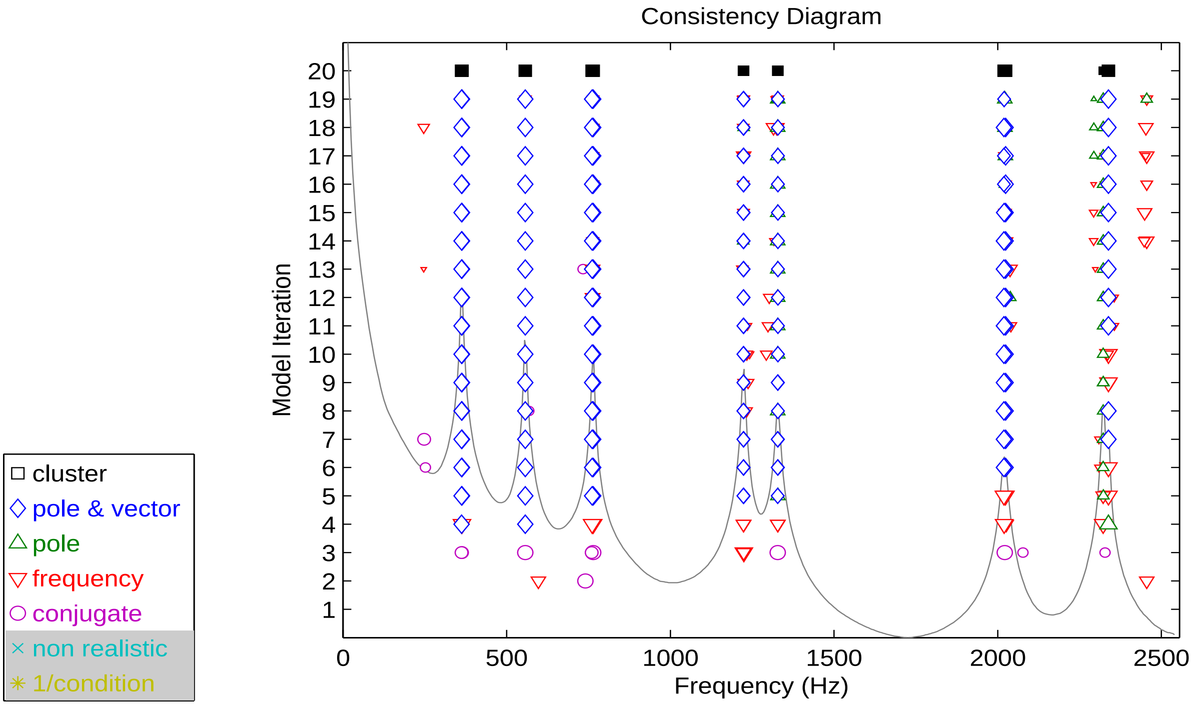

Figure 16. Consistency Diagram with Final Autonomous Estimates

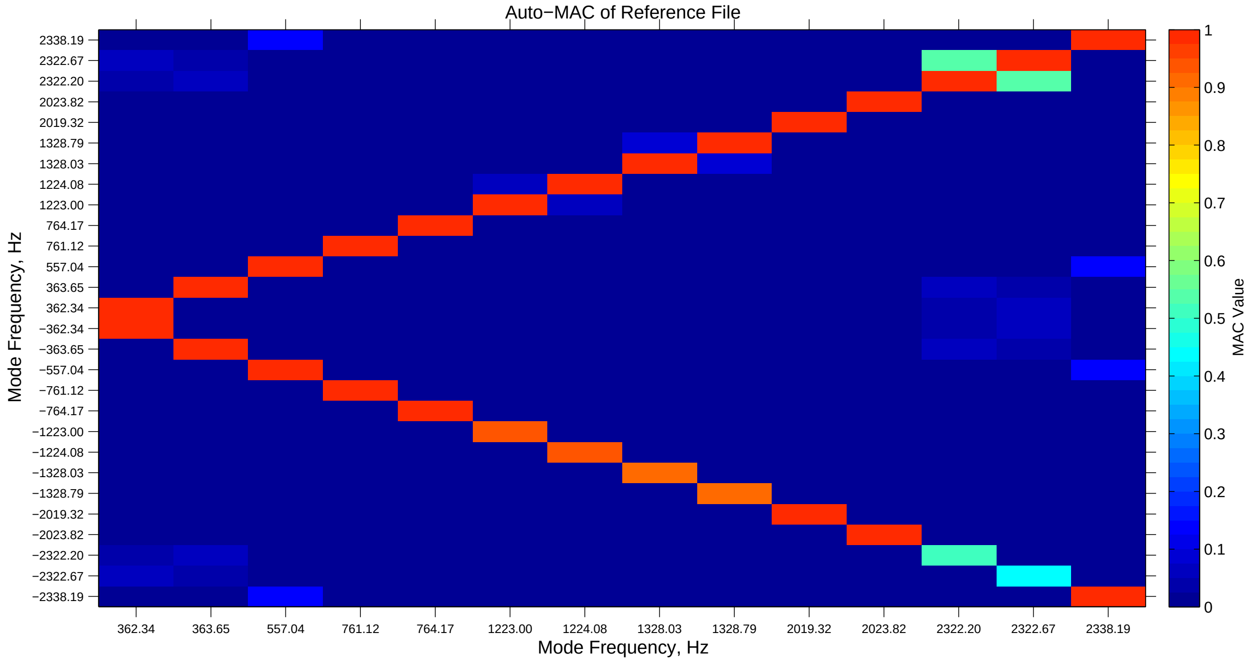

Figure 17. Auto MAC of Final Autonomous Estimates - Conjugate Poles

Once again, Figure 17 has been found to be a useful indicator when estimating modal parameters for a dominantly normal mode system. The Auto MAC plot of the estimated modal vectors shown in Figure 17 shows a nearly perfect correlation between the vectors associated with the positive frequency and the vector associated with the estimate of its negative frequency (conjugate pair).

5.2.2 Application Example: Bridge Field Test Data

The next example of the autonomous UMPA method is performed in the field on a small highway bridge. The FRF data in this case has 55 inputs and 15 responses. This data set has been particularly troublesome when analyzed by any method available and is shown as a significant implementation of the autonomous modal parameter estimation procedure. The autonomous results for this case are as good or better compared to any other solution utilized in the past, as measured by reasonable estimates of frequency and damping and dominantly normal modes.

In this case, a Rational Fraction Polynomial with complex z frequency mapping (RFP-Z) method is used (similar to PLSCF and PolyMAX ® but, for every possible pole estimated over a model order range from 2 to 20, a complete long dimension modal vector was estimated so that sufficient spatial information is available for sorting out the consistent solutions. For this case, the following thresholds and control parameters were used:

- Lowest order coefficient matrix normalization.
- Pole density threshold (4 and above).
- Pole weighted vector of model order 10.
- Pole weighted MAC threshold (0.8 and above).
- Cluster size threshold (3 and above).
- Cluster identification threshold (0.8 and above).

This case demonstrates that, even with clear stabilization diagram techniques, the consistency diagram can get fairly difficult to interpret. Nonetheless, in this application of the autonomous modal parameter estimation procedure, it is possible to identify the modal parameters with little trouble.

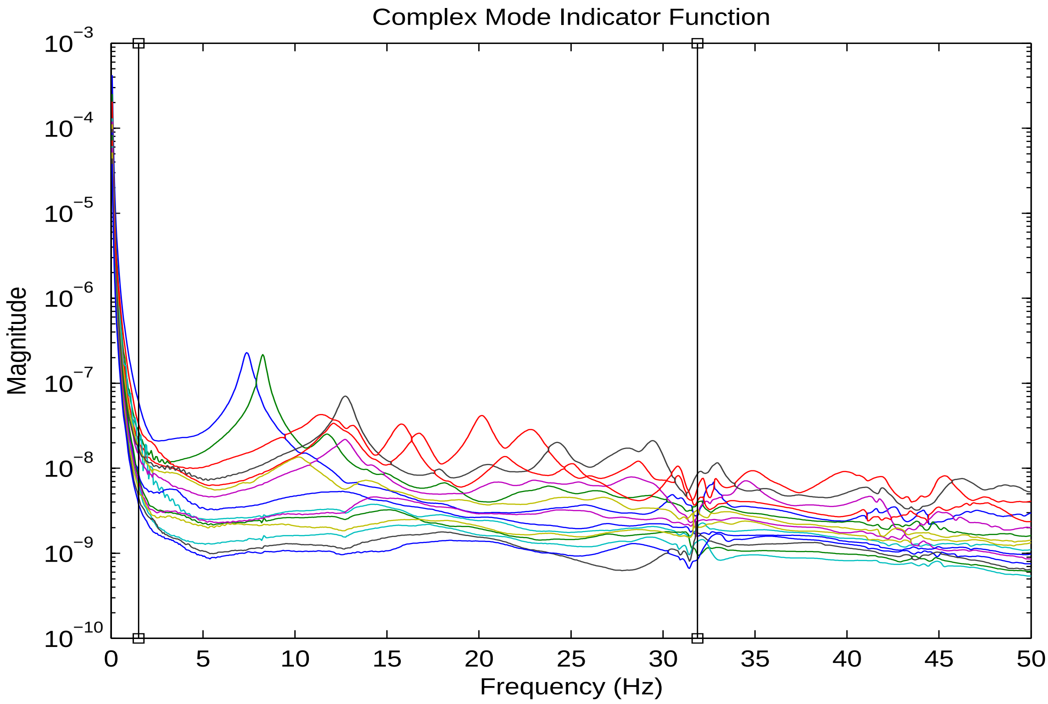

Figure 18. Complex Mode Indicator Function (CMIF)

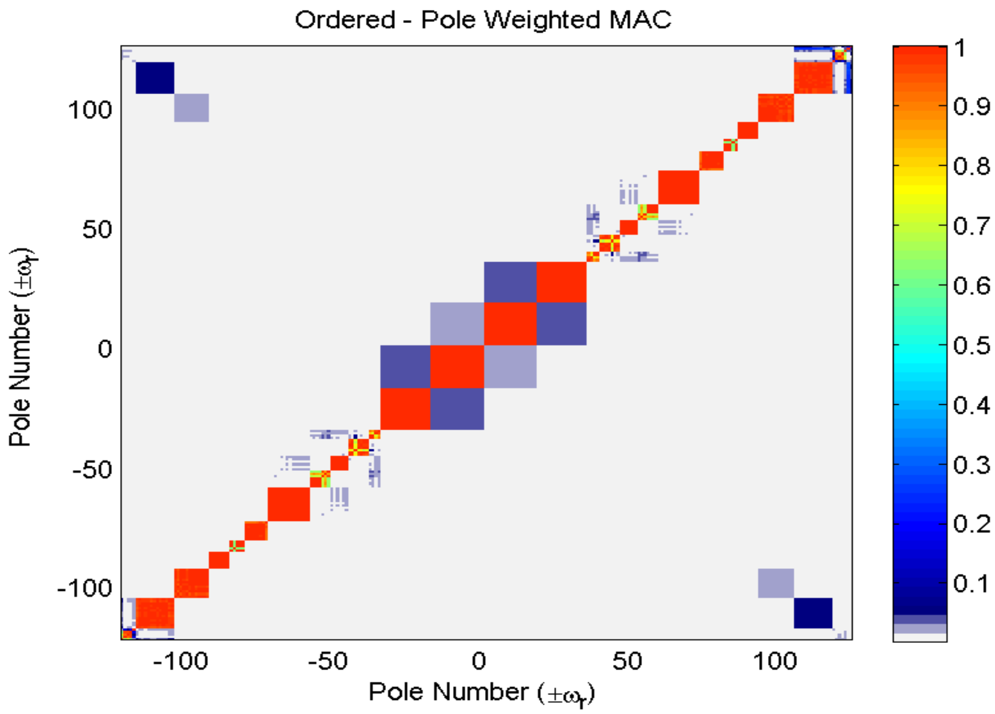

Figure 19. MAC-Tenth Order Pole Weighted Vectors, No Threshold

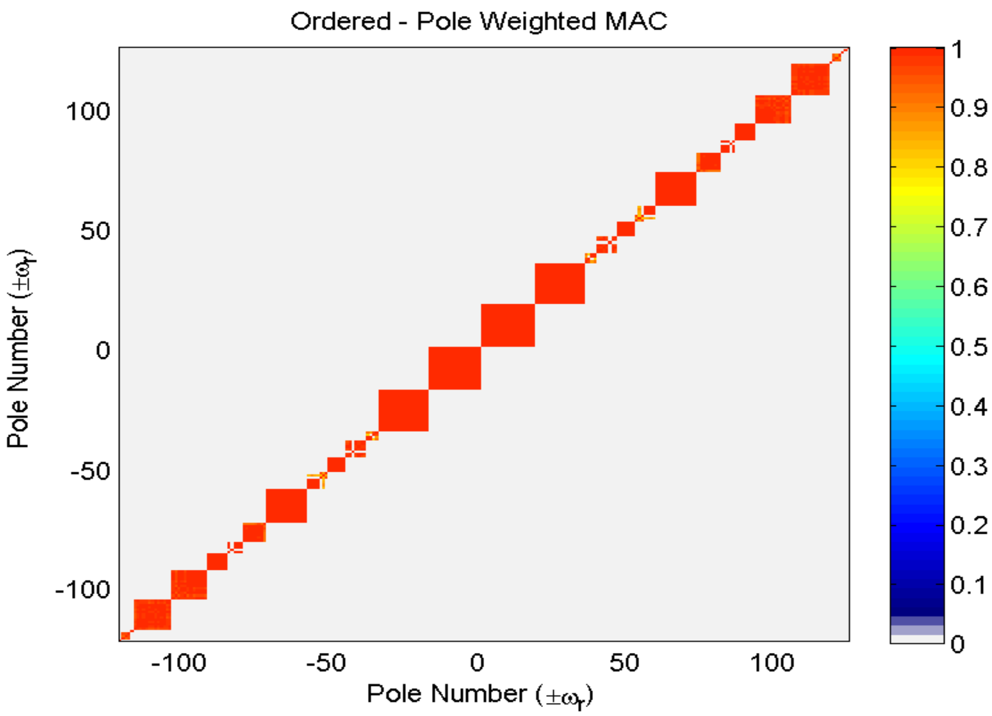

Figure 20. MAC-Tenth Order Pole Weighted Vectors, Above Threshold

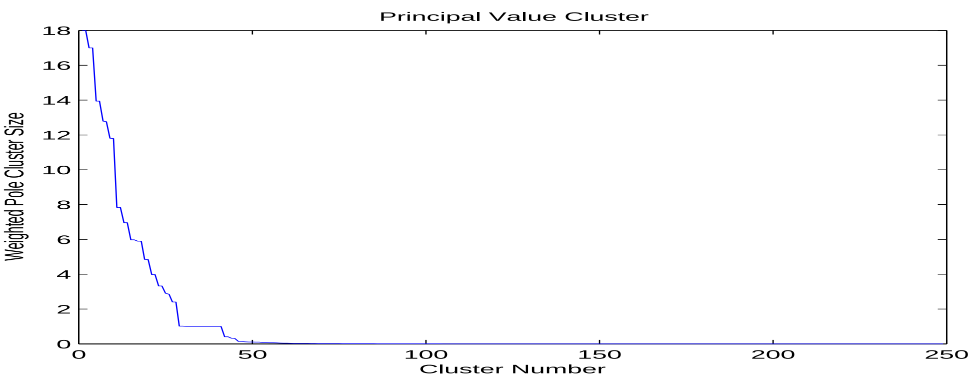

Figure 21. Principal Values of Clusters of Pole Weighted Vectors

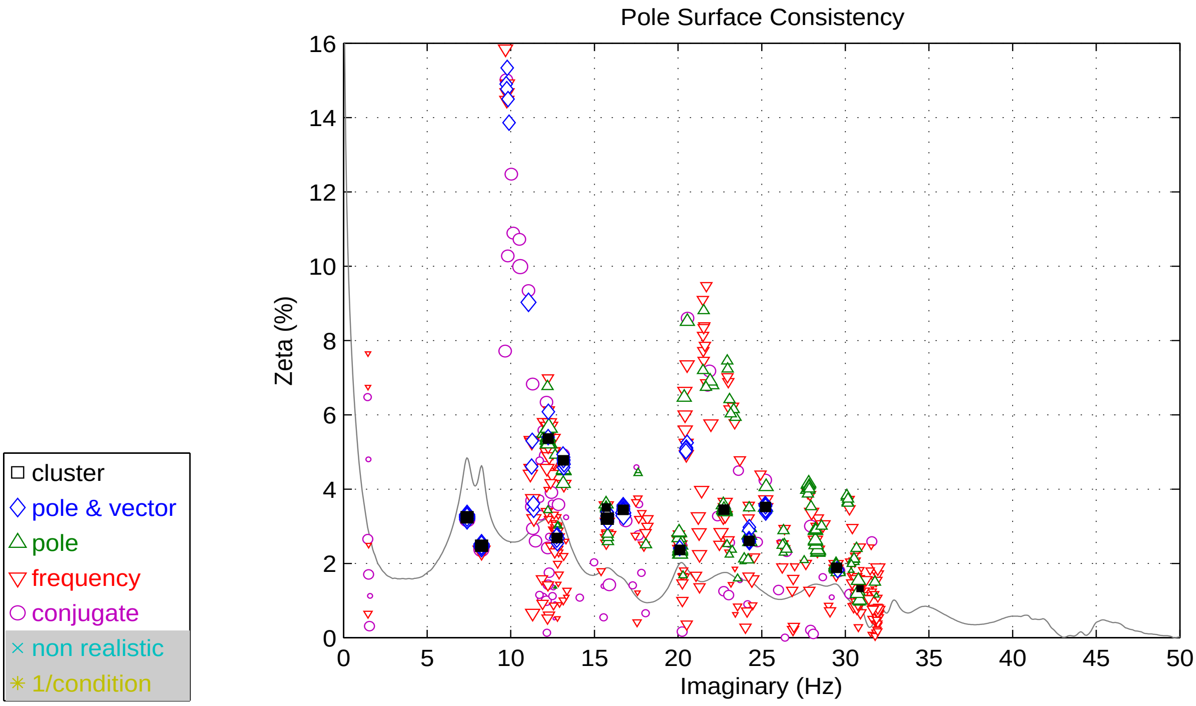

Figure 22. Pole Surface Consistency Clusters with Final Autonomous Estimates

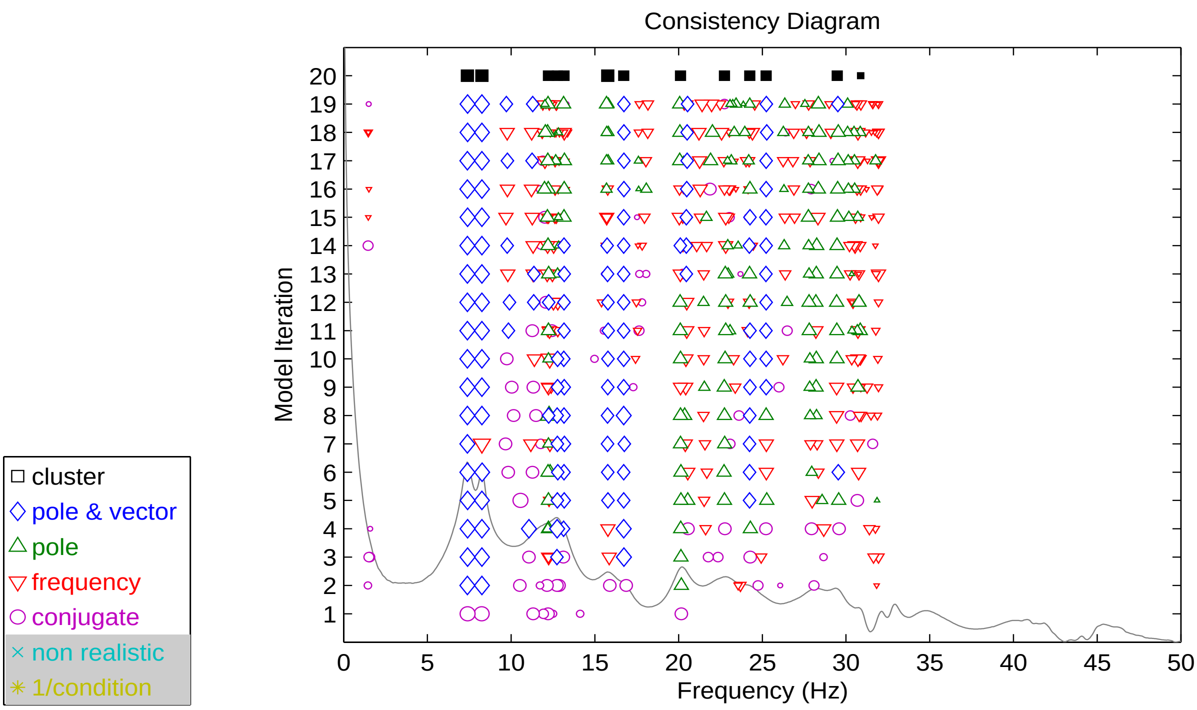

Figure 23. Consistency Diagram with Final Autonomous Estimates

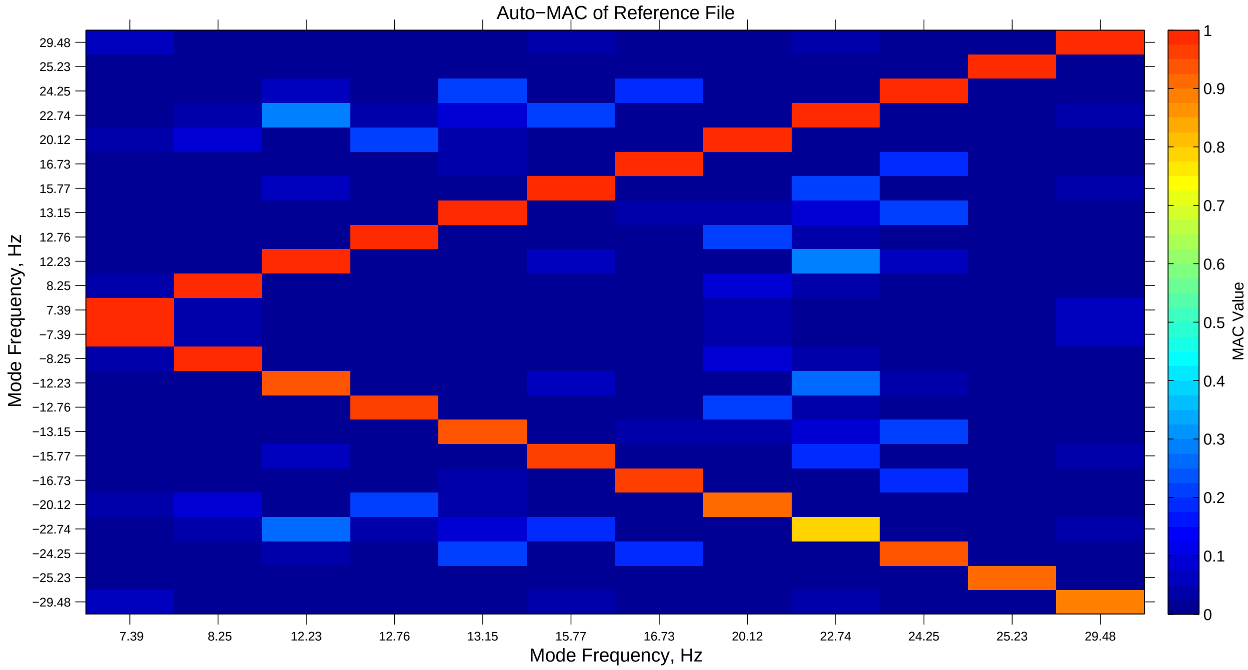

Figure 24. Auto MAC of Final Autonomous Estimates - Conjugate Poles

5.2.3 Application Example: C-Plate Laboratory Test Data with Short Dimension Base Vector

The next example of the autonomous UMPA method is performed on the same, circular plate laboratory test object. The FRF data is the same as the previous example and the Polyreference Time Domain (PTD) method is used but only the short dimension vectors are used for every possible pole estimated over a model order range from 2 to 20. This means that the spatial information (base vector of length 7) available for sorting out the consistent solutions will be compromised. For this case, the following thresholds and control parameters were used:

- Lowest order coefficient matrix normalization.
- Pole density threshold (4 and above).
- Pole weighted vector of model order 10.
- Pole weighted MAC threshold (0.8 and above).
- Cluster size threshold (4 and above).
- Cluster identification threshold (0.8 and above).

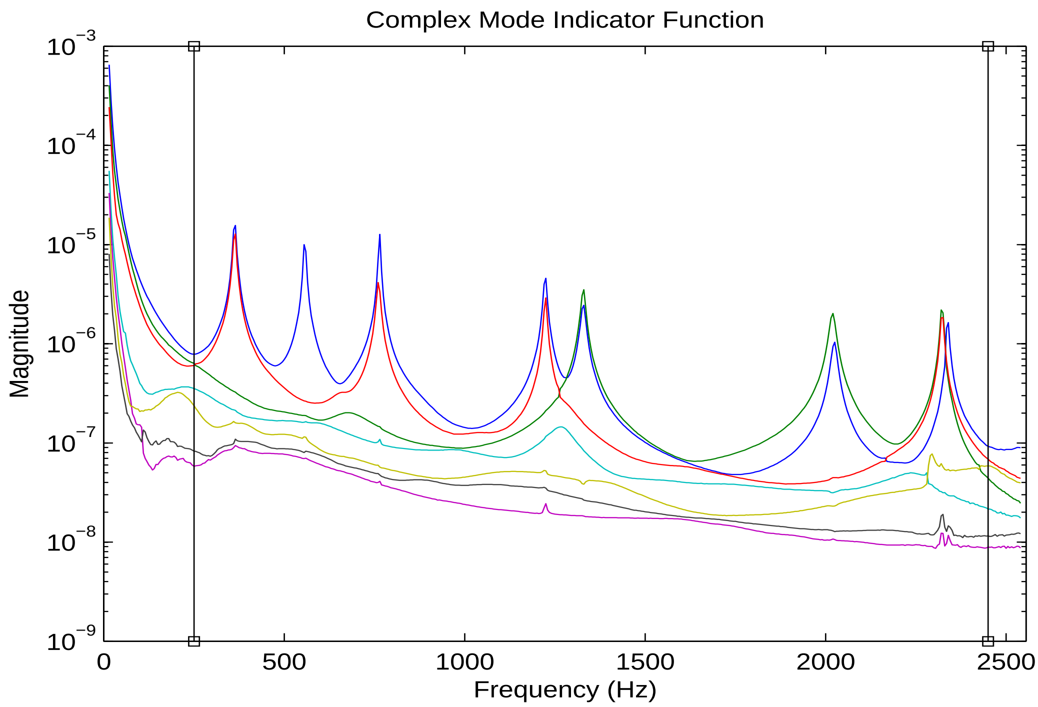

Figure 25. Complex Mode Indicator Function (CMIF)

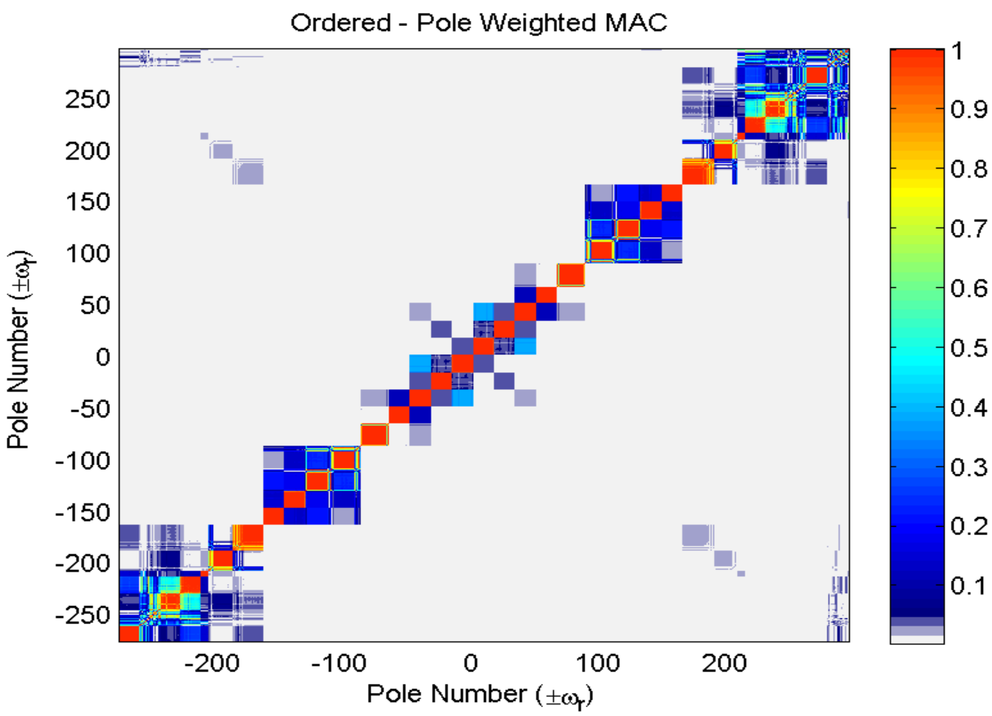

Figure 26. MAC-Tenth Order Pole Weighted Vectors, No Threshold

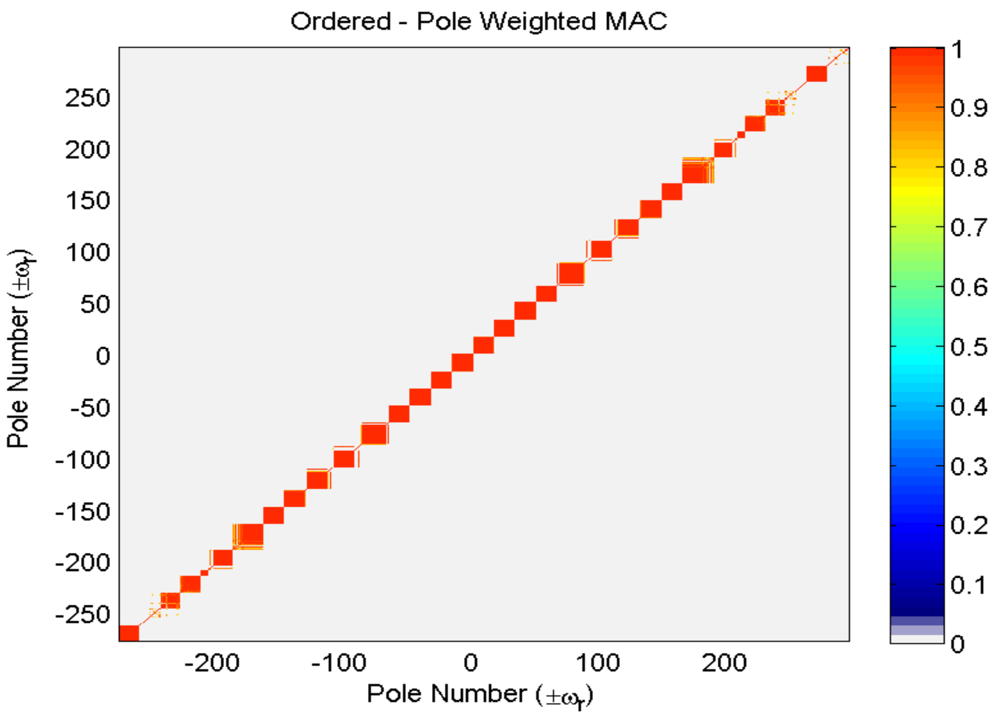

Figure 27. MAC-Tenth Order Pole Weighted Vectors, Above Threshold

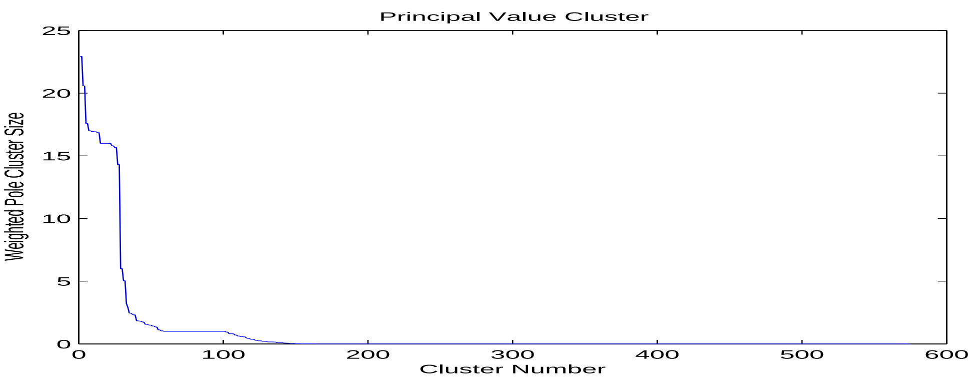

Figure 28. Principal Values of Clusters of Pole Weighted Vectors

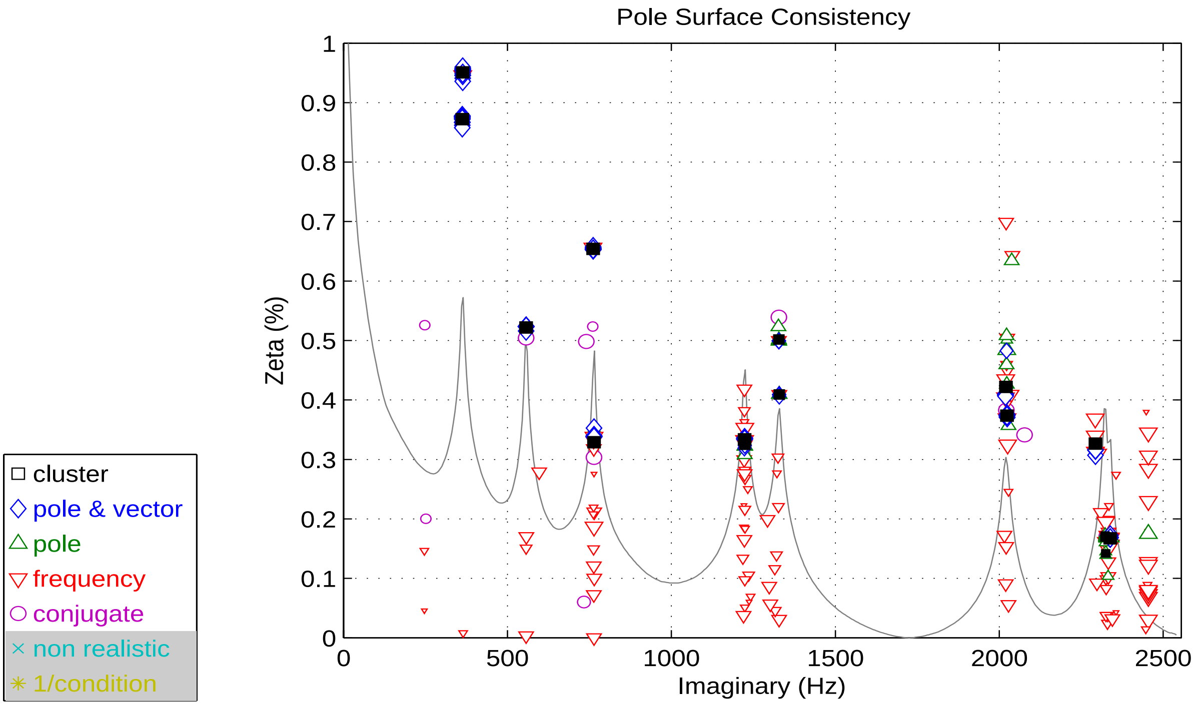

Figure 29. Pole Surface Consistency Clusters with Final Autonomous Estimates

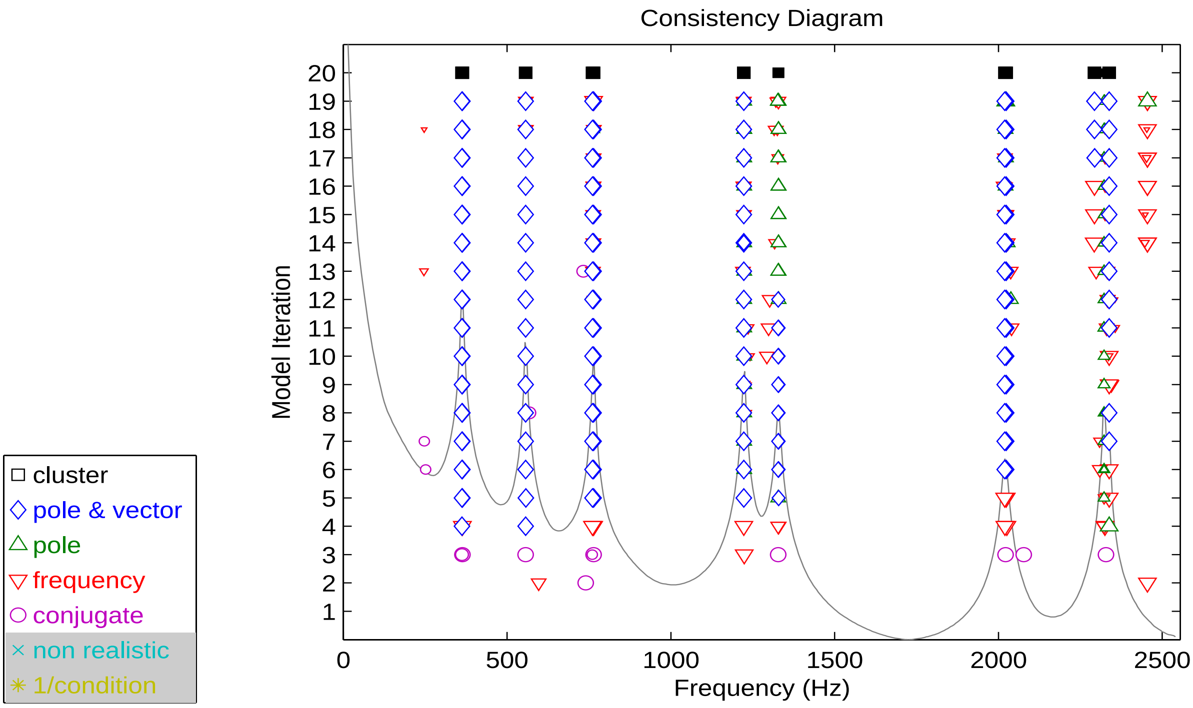

Figure 30. Consistency Diagram with Final Autonomous Estimates

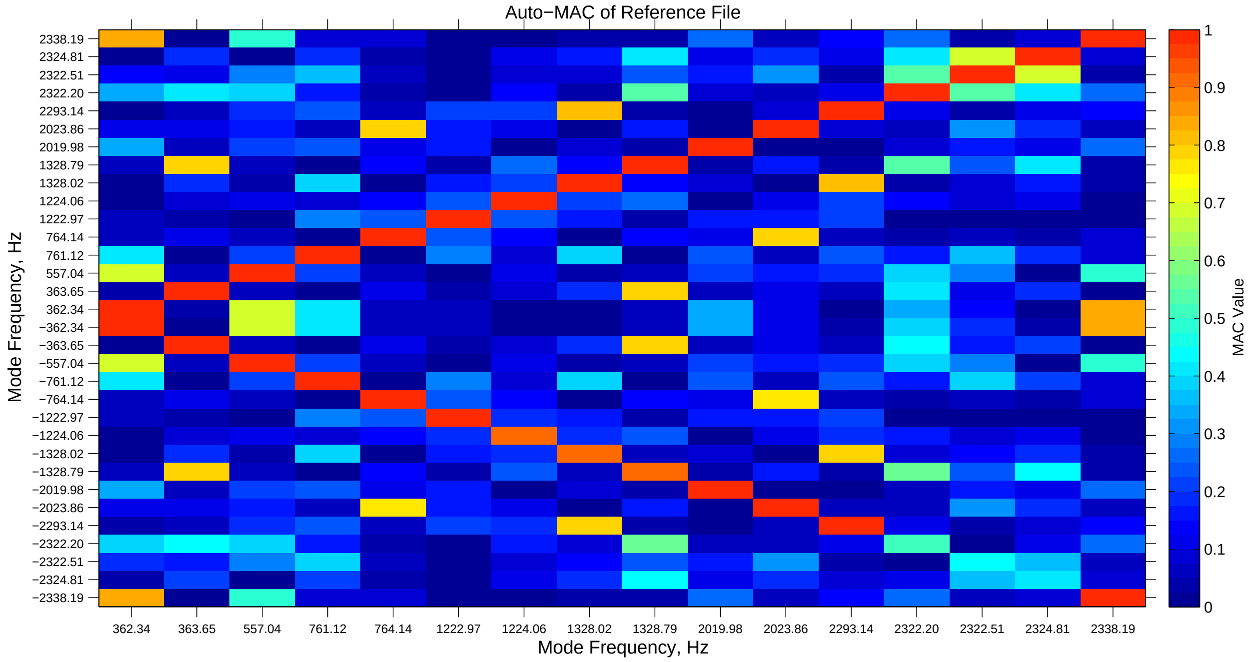

Figure 31. Auto MAC of Final Autonomous Estimates - Conjugate Poles

While the results are certainly acceptable for this case, the Auto MAC plot in Figure 31 shows some additional clutter and the conjugate properties of the higher frequency modes do not match as well as in the previous, long dimension base vector example.

5.2.4 Application Example: C-Plate Laboratory Test Data with Sieved, Short Dimension Base Vector

The next example of the autonomous UMPA method is performed on the same, circular plate laboratory test object. The FRF data is the same as the previous C-Plate examples and the Polyreference Time Domain (PTD) method is used but only the short dimension vectors are used for every possible pole estimated over a model order range from 2 to 20. This short dimension is further sieved to only two references. This means that the spatial information (base vector of length 2) available for sorting out the consistent solutions will be further compromised. For this case, the following thresholds and control parameters were used:

- Lowest order coefficient matrix normalization.
- Pole density threshold (4 and above).
- Pole weighted vector of model order 10.
- Pole weighted MAC threshold (0.8 and above).
- Cluster size threshold (4 and above).
- Cluster identification threshold (0.8 and above).

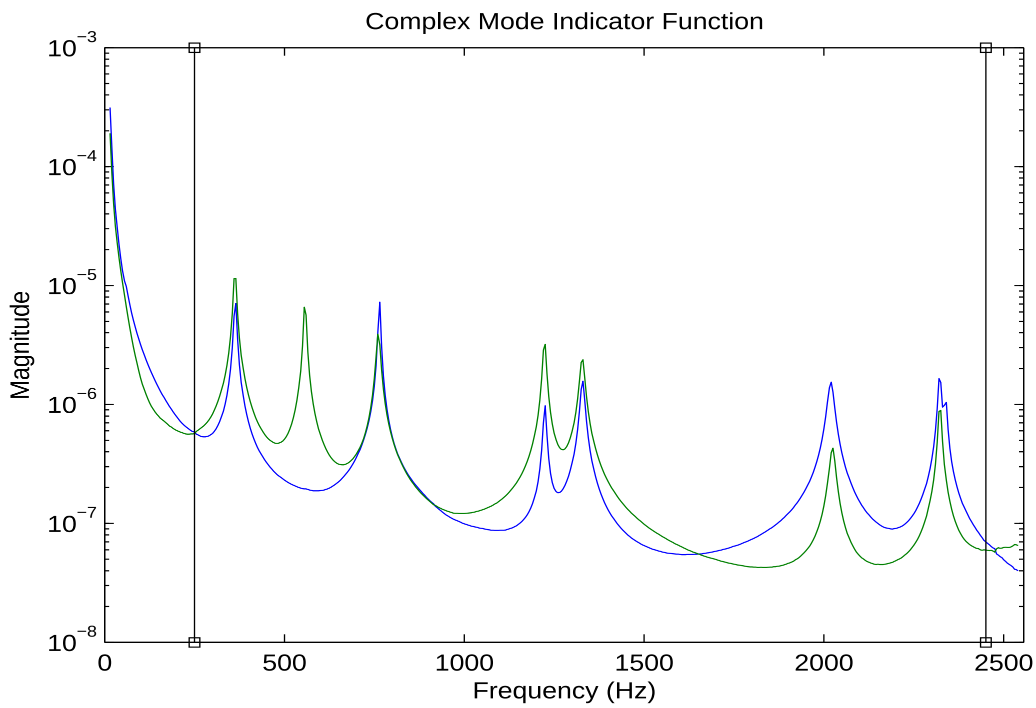

Figure 32. Complex Mode Indicator Function (CMIF)

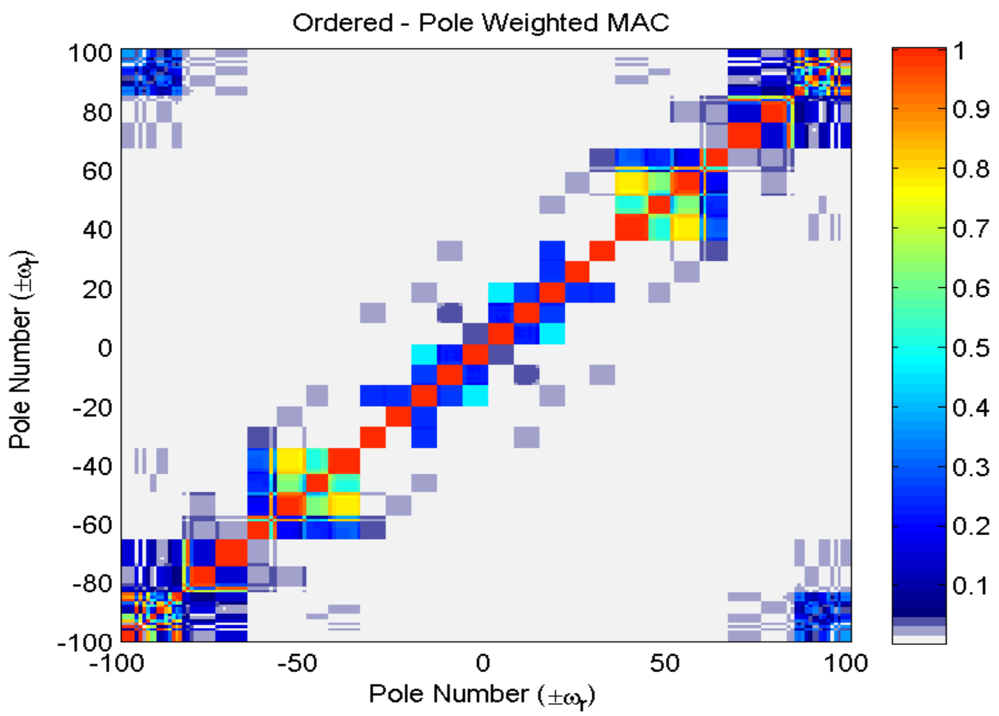

Figure 33. MAC-Tenth Order Pole Weighted Vectors, No Threshold

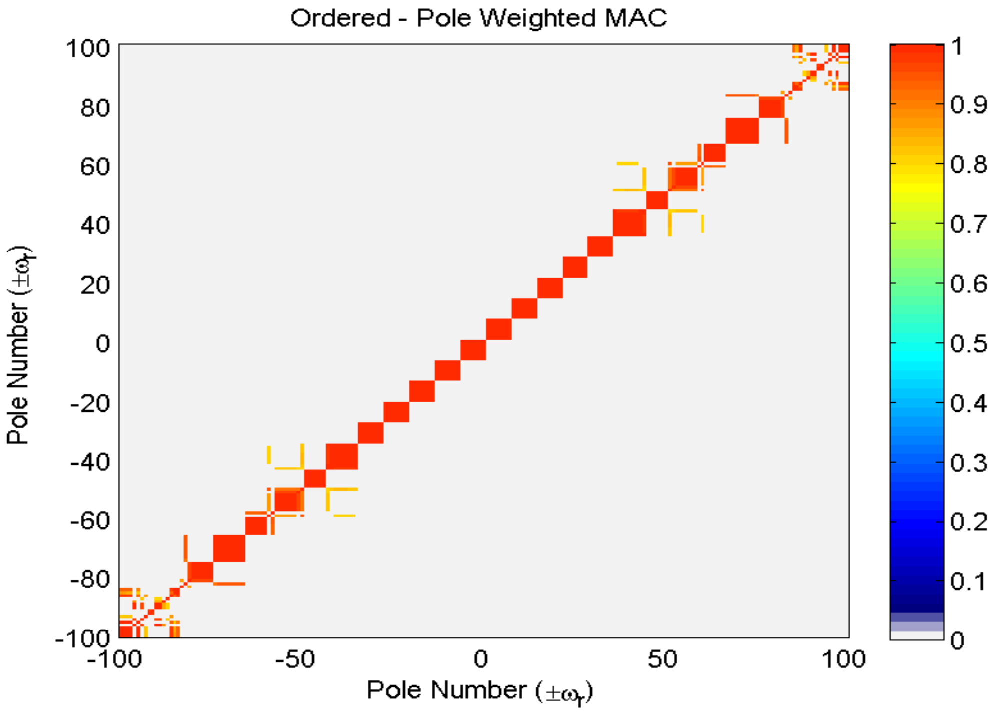

Figure 34. MAC-Tenth Order Pole Weighted Vectors, Above Threshold

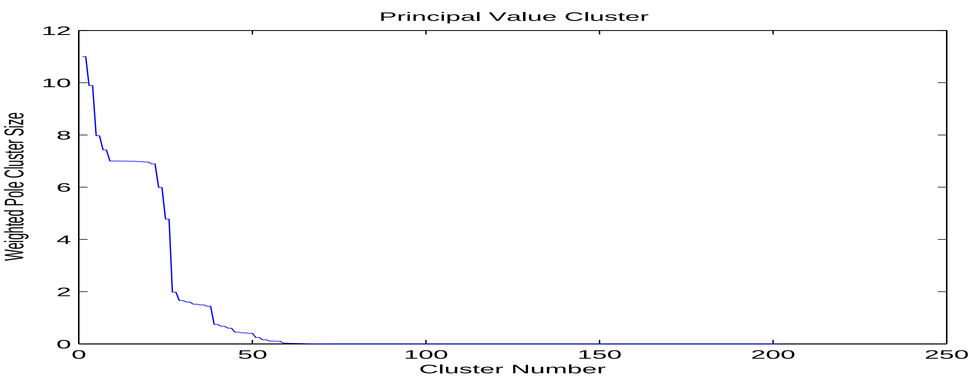

Figure 35. Principal Values of Clusters of Pole Weighted Vectors

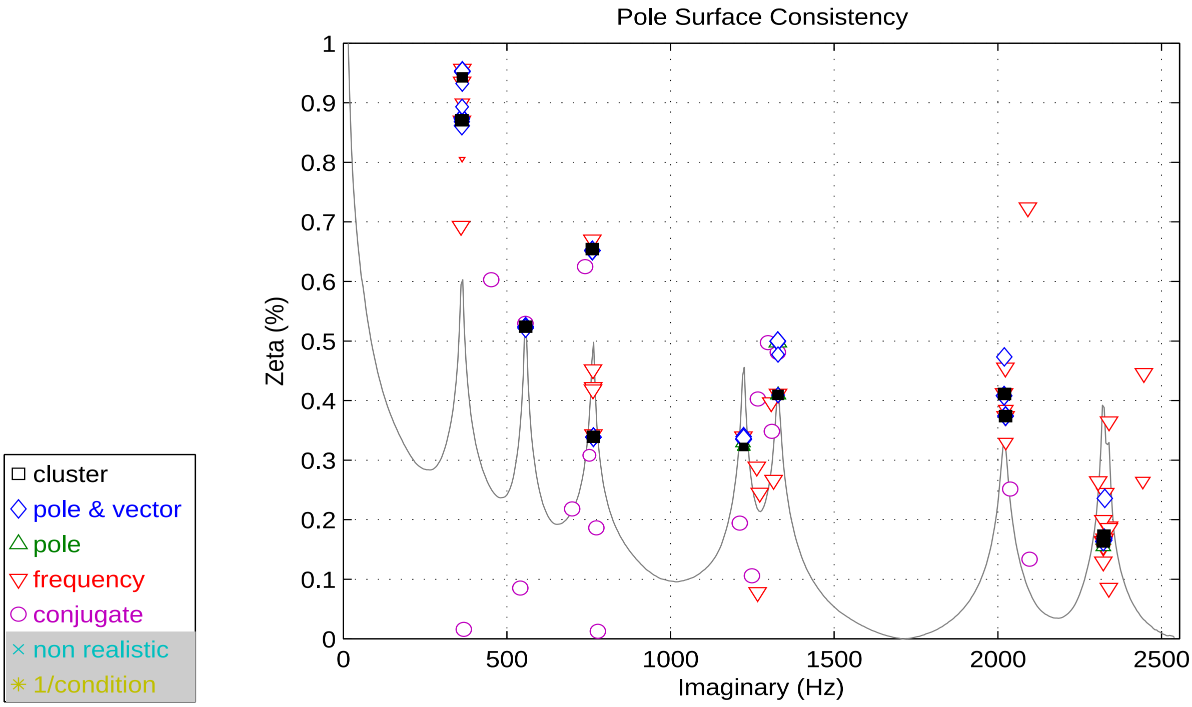

Figure 36. Pole Surface Consistency Clusters with Final Autonomous Estimates

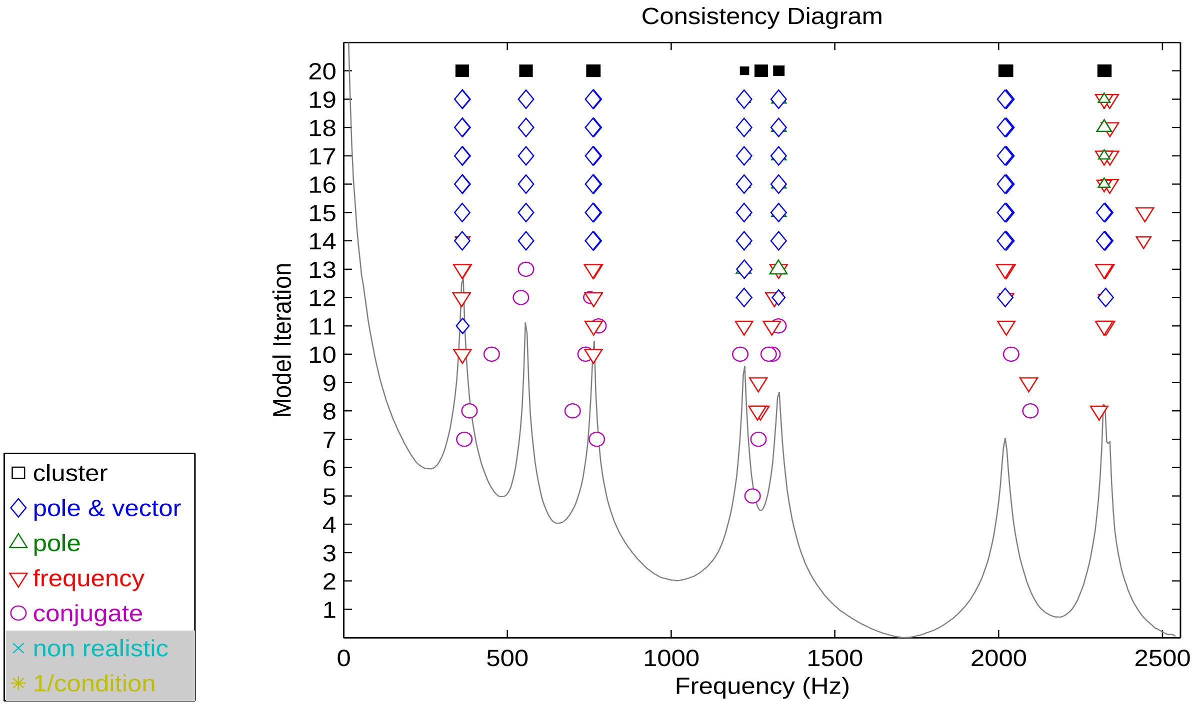

Figure 37. Consistency Diagram with Final Autonomous Estimates

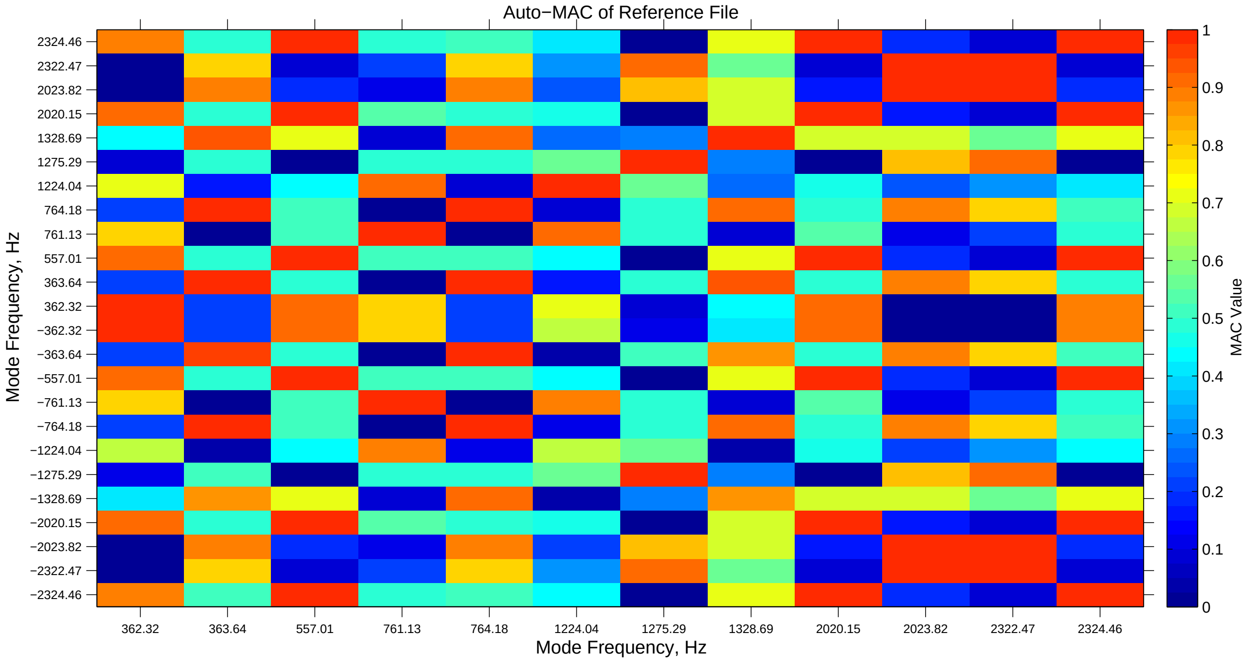

Figure 38. Auto MAC of Final Autonomous Estimates - Conjugate Poles

The results for this case are further degraded as noted by the Auto MAC plot in Figure 38 shows some additional clutter and the conjugate properties of the higher frequency modes do not match as well as in the previous, long dimension base vector example.

5.2.5 Application Example: C-Plate Laboratory Test Data with Sieved, Short Dimension Base Vector

The final example of the autonomous UMPA method is performed on the same, circular plate laboratory test object. The FRF data is the same as the previous C-Plate examples and the Polyreference Time Domain (PTD) method is used but only the short dimension vectors are used for every possible pole estimated over a model order range from 2 to 20. This short dimension is further sieved to only two references. This means that the spatial information (base vector of length 2) available for sorting out the consistent solutions will be compromised. In order to try to compensate for this extremely short base vector, the order of the pole weighted vector is increased to 50. For this case, the following thresholds and control parameters were used:

- Lowest order coefficient matrix normalization.
- Pole density threshold (4 and above).
- Pole weighted vector of model order 50.
- Pole weighted MAC threshold (0.8 and above).
- Cluster size threshold (4 and above).
- Cluster identification threshold (0.8 and above).

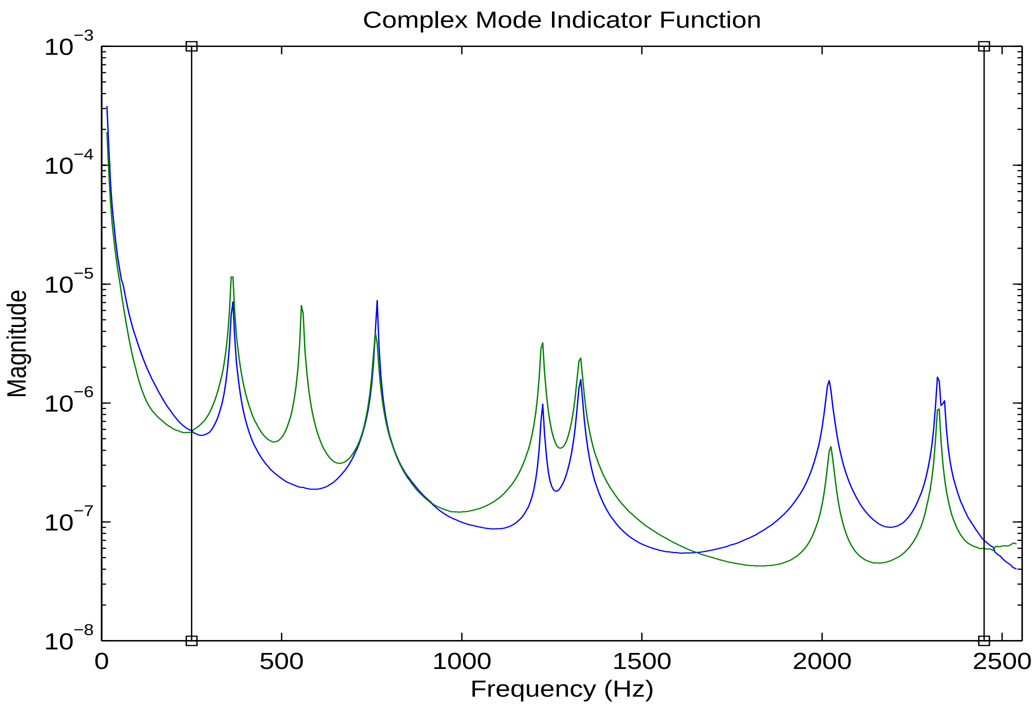

Figure 39. Complex Mode Indicator Function (CMIF)

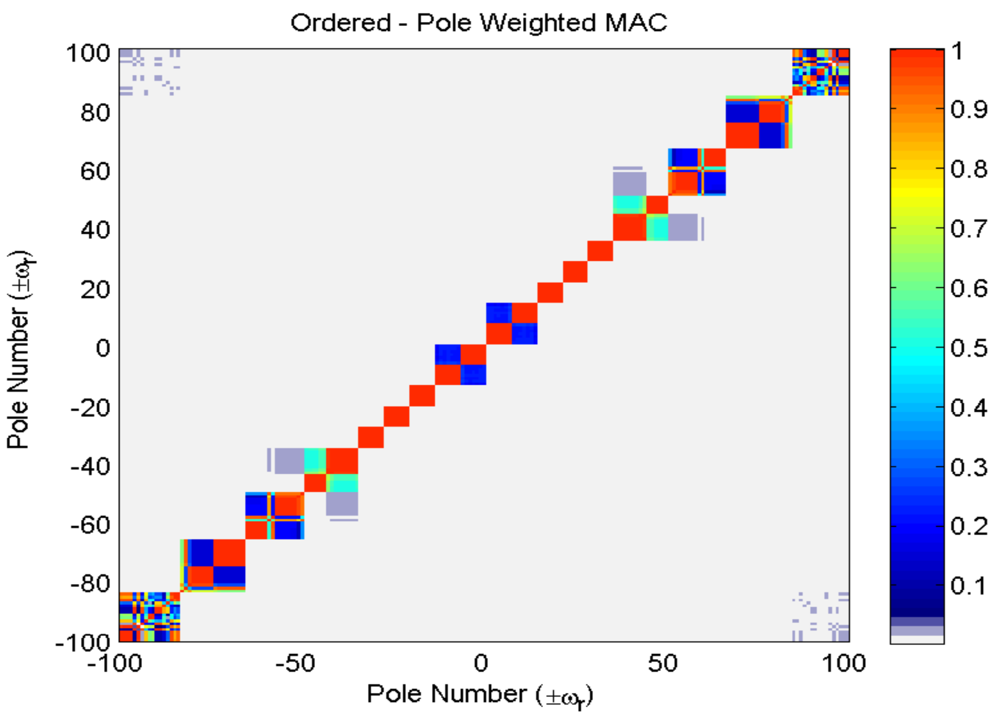

Figure 40. MAC-Fiftieth Order Pole Weighted Vectors, No Threshold

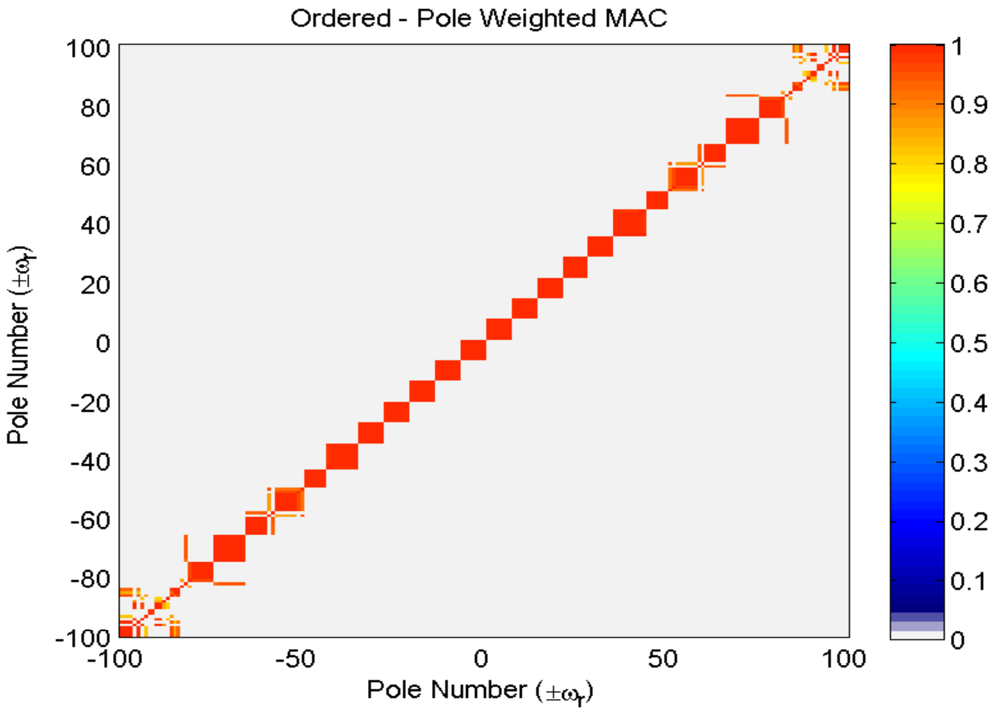

Figure 41. MAC-Fiftieth Order Pole Weighted Vectors, Above Threshold

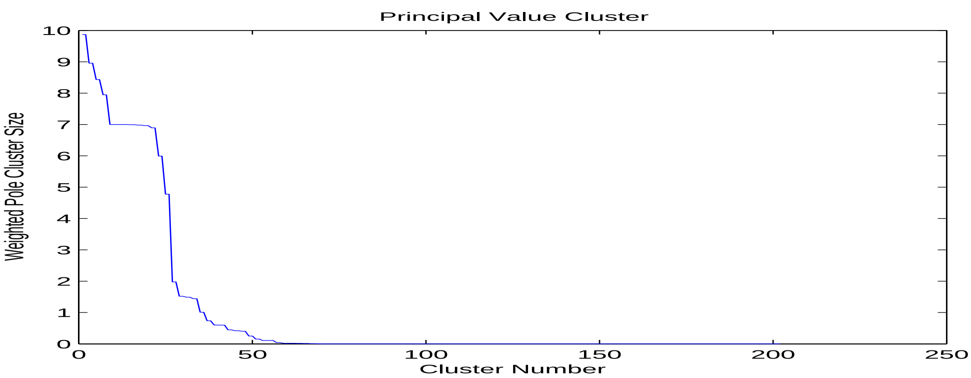

Figure 42. Principal Values of Clusters of Pole Weighted Vectors

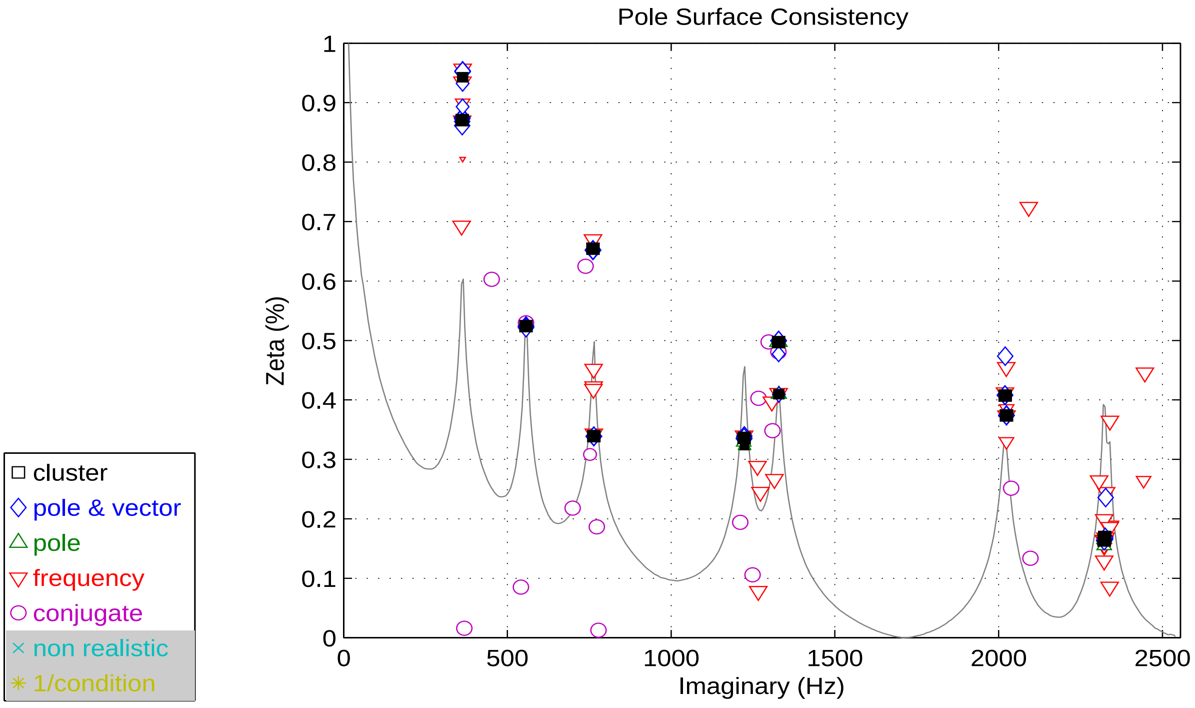

Figure 43. Pole Surface Consistency Clusters with Final Autonomous Estimates

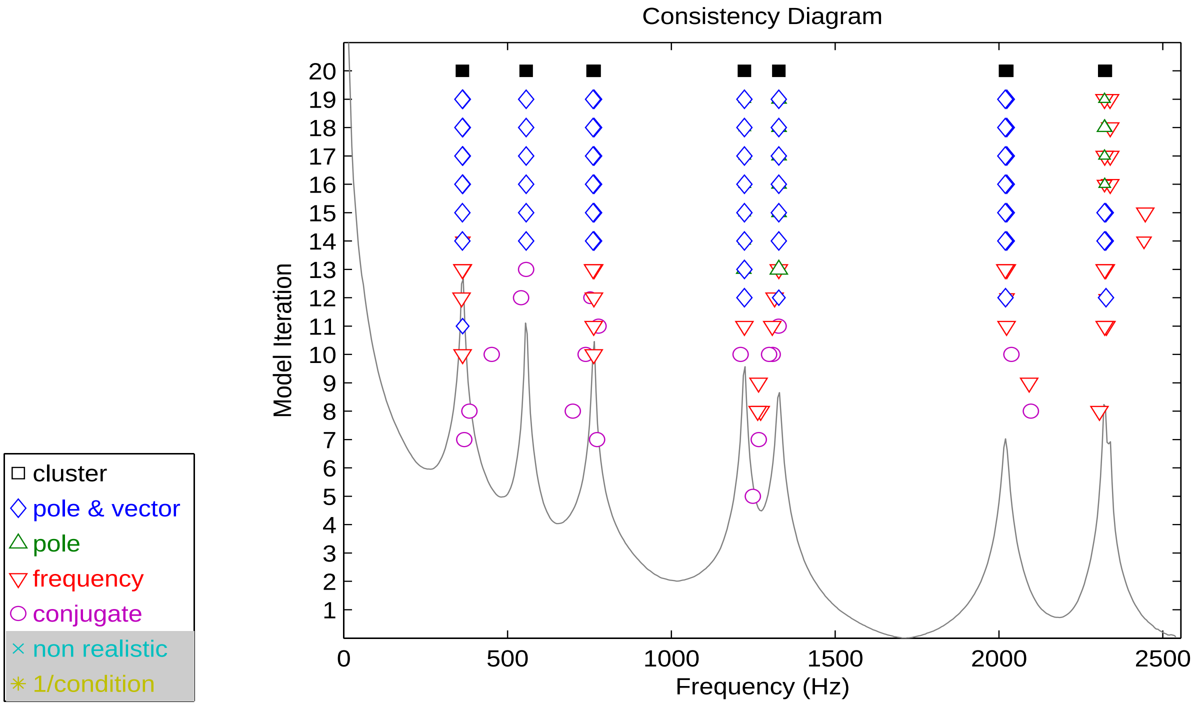

Figure 44. Consistency Diagram with Final Autonomous Estimates

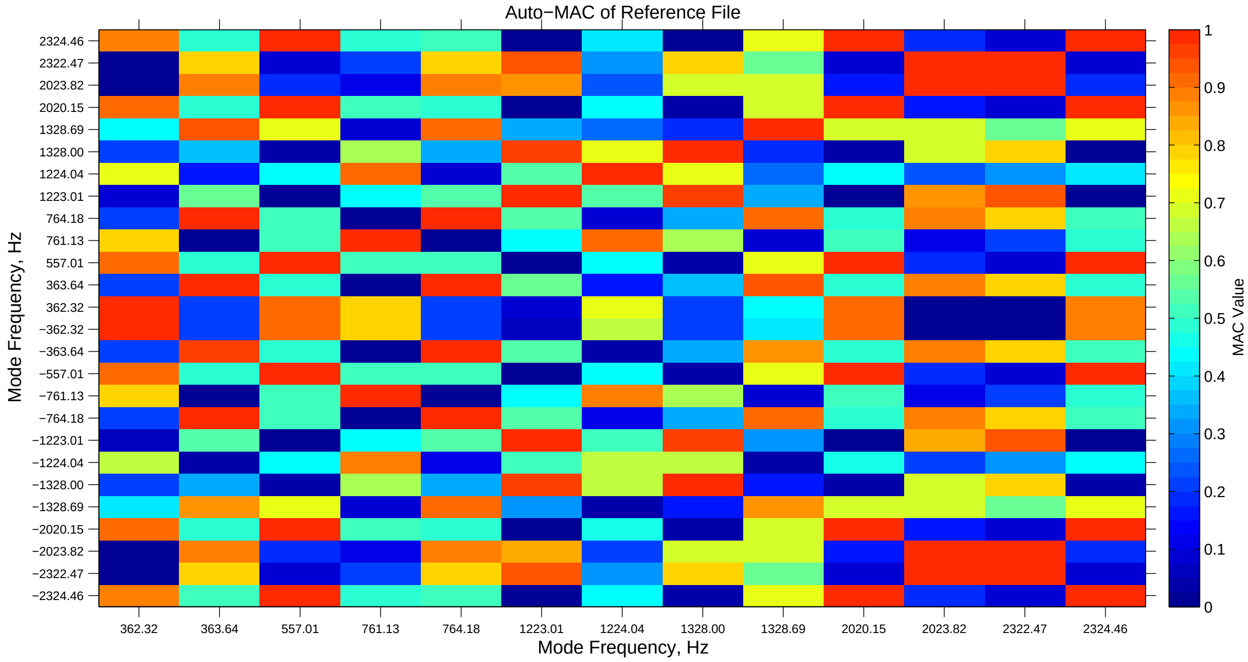

Figure 45. Auto MAC of Final Autonomous Estimates - Conjugate Poles

The results for this case are very similar to the previous example as noted by the Auto MAC plot in Figure 45. This Auto MAC plot shows some additional clutter and the conjugate properties of the higher frequency modes do not match as well as in the previous, long dimension base vector example. The conclusion at this point would be to always utilize the long base vector in order to get the most reliable result.

6. Summary and Future Work

In this paper, two autonomous modal parameter identification procedures have been presented along with a review of the theory applicable to the development of the autonomous procedures.

The first autonomous procedure involves a relatively simple method of correlating the two potential normalization solutions of a traditional low order, long basis algorithm. The novelty in this case is the enhanced sorting capability of using extended state basis vectors in the MAC correlation process.

The second autonomous method represents the important extension of correlating the extended basis vector to multiple (more than two) sets of solutions involving either long or short basis vectors. This technique has been shown to be general and applicable to most standard commercially available algorithms. The application of the technique to several laboratory and real world structures has been shown.

With the advent of more computationally powerful computers and sufficient memory, it has become practical to evaluate sets of solutions involving thousands of modal parameter estimates and to extract the common information from those sets. Both autonomous procedures give very acceptable results, in some cases superior results, in a fraction of the time required for an experienced user to get the same result.

However, it is important to reiterate that the use of these autonomous procedures or *wizard* tools by users with limited experience is probably not yet appropriate. Such tools are most appropriately used by analysts with the experience to accurately judge the quality of the parameter solutions identified.

Future work on these methods planned by the authors include: combining the solutions from different

UMPA methods (PTD plus RFP-Z plus ERA, for example), further evaluating the sensitivity of control parameters and thresholds, adding statistical descriptors for each estimated set of modal parameters and investigating more application cases such as purely academic, theoretical examples as well as difficult sets of measured, real-world data cases.

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