Integrating Multiple Algorithms In Autonomous Modal Parameter Estimation

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Abstract

Recent work with autonomous modal parameter estimation has shown great promise in the quality of the modal parameter estimation results when compared to results from experienced user interaction using traditional methods. Current research with the Common Statistical Subspace Autonomous Mode Identification (CSSAMI) procedure involves the integration of multiple modal parameter estimation algorithms into the autonomous procedure. The current work uses possible solutions from different traditional methods like Polyreference Time Domain (PTD), Eigensystem Realization Algorithm (ERA) and Polyreference Frequency Domain (PFD) that are combined in the autonomous procedure to yield one consistent set of modal parameter solutions. This final, consistent set of modal parameters is identifiable due to the combination of temporal information (the complex modal frequency) and the spatial information (the modal vectors) in a Z domain state vector of relatively high order (5-10). Since this Z domain state vector has the complex modal frequency and the modal vector as embedded content, sorting consistent estimates from hundreds or thousands of possible solutions is now relatively trivial based upon the use of a state vector involving spatial information.

Keywords: Autonomous, Modal Parameter Estimation, Pole Weighted Vector, State Vector, Experimental Structural Dynamics

Nomenclature

N_i = Number of inputs.
N_o = Number of outputs.
N_S = Short dimension size.
N_L = Long dimension size.
N = Number of vectors in cluster.
ω_i = Discrete frequency (rad/sec).
$[H(\omega_i)]$ = FRF matrix ($N_o \times N_i$)).
r = Mode number.

λ_r = S domain polynomial root.
λ_r = Complex modal frequency (rad/sec).
$\lambda_r = \sigma_r + j\ \omega_r$
σ_r = Modal damping.
ω_r = Damped natural frequency.
z_r = Z domain polynomial root.
$\{\psi_r\}$ = Base vector (modal vector).
$\{\phi_r\}$ = Pole weighted base vector (state vector).

1. Introduction

The desire to estimate modal parameters automatically, once a set or multiple sets of test data are acquired, has been a subject of great interest for more than 40 years. Even in the 1960s, when modal testing was limited to analog test methods, several researchers were exploring the idea of an automated test procedure for determining modal parameters [1-3]. Today, with the increased memory and compute power of current computers used to process test data, an automated or autonomous, modal parameter estimation procedure is entirely possible and is being evaluated by numerous researchers and users.

Before proceeding with a discussion of how multiple modal parameter estimation algorithms can be combined into autonomous modal parameter estimation, some discussion of the current autonomous modal parameter estimation procedure is required. In general, autonomous modal parameter estimation refers to an automated procedure that is applied to a modal parameter estimation algorithm so that no user interaction is required once the process is initiated. This typically involves setting a number of parameters or thresholds that are used to guide the process in order to exclude solutions that are not acceptable to the user. When the procedure finishes, a set of modal parameters is identified that can then be reduced or expanded if necessary. The goal is that no further reduction, expansion or interaction with the process will be required.

For the purposes of further discussion, the autonomous modal parameter estimation procedure is simply an efficient mechanism for sorting a very large number of solutions into a final set of solutions that satisfies a set of criteria and thresholds that are acceptable to the user. When multiple modal parameter estimation algorithms are combined into a single autonomous procedure, this yields more estimates of the modal parameters which contribute to a statistically more significant result. Currently, the user of autonomous modal parameter estimation is assumed to be very experienced and is using autonomous modal parameter estimation as a sophisticated tool to highlight the most likely solutions based upon statistics. The experienced user will realize that the final solutions may include unrealistic solutions or non-optimal solutions and further evaluation will be required.

2. Background

In order to discuss the impact and use of multiple modal parameter estimation algotrithms in autonomous modal parameter estimation, the importance of spatial information to the solution procedure is critical. Therefore, some background is needed to clarify terminology and methodology. This background has been provided in previous papers [4-7] and will only be highlighted here in terms of spatial information, modal parameter estimation and autonomous modal parameter estimation.

2.1 Spatial Information

Spatial information, with respect to experimental modal parameter estimation, refers to the vector information and dimension associated with the inputs and outputs of the experimental test. Essentially, this represents the locations of the sensors in the experimental test. It is important to recognize that an experimental test should always include multiple inputs and outputs in order to clearly estimate different modal vectors and to resolve modal vectors when the complex natural frequencies are close, what is called repeated or pseudo-repeated roots.

Since the data matrix, normally involving frequency response functions (FRF) or impulse response functions (IRF), is considered to be symmetric or reciprocal, the data matrix can be transposed, switching the effective meaning of the row and column index with respect to the physical inputs and outputs.

$$[\, H(\omega_i) \,]_{N_o \times N_i} = [\, H(\omega_i) \,]^T_{N_i \times N_o} \tag{1}$$

Since many modal parameter estimation algorithms are developed on the basis of either the number of inputs (N_i) or the number of outputs (N_o), assuming that one or the other is larger based upon test method, some nomenclature conventions are required for ease of further discussion. In terms of the modal parameter estimation algorithms, it is more important to recognize whether the algorithm develops the solution on the basis of the larger (N_L) of N_i or N_o, or the smaller (N_S) of N_i or N_o, dimension of the experimental data. For this reason, the terminology of *long* (larger of N_i or N_o) dimension or *short* (smaller of N_i or N_o) dimension is easier to understand without confusion.

Therefore, the nomenclature of the number of outputs (N_o) and number of inputs (N_i) has been replaced by the length of the long dimension of the data matrix (N_L) and the length of the short dimension (N_S) regardless of which dimension refers to the physical output or input. This means that the above reciprocity relationship can be restated as:

$$[\, H(\omega_i) \,]_{N_L \times N_S} = [\, H(\omega_i) \,]^T_{N_S \times N_L} \tag{2}$$

Note that the reciprocity relationships embedded in Equation (1) and (2) are a function of the common degrees of freedom (DOFs) in the short and long dimensions. If there are no common DOFs, there are no reciprocity relationships and the data requirement for modern modal parameter estimation algorithms (multiple references) will not be met. Nevertheless, the importance of Equation (1) and (2) is that the dimensions of the FRF matrix can be transposed as needed to fit the requirement of specific modal parameter estimation algorithms. This impacts the size of the square matrix coefficients in the matrix coefficient, polynomal equation and the length of the associated modal (base) vector.

2.2 Autonomous Modal Parameter Estimation

The interest in automatic modal parameter estimation methods has been documented in the literature since at least the mid 1960s when the primary modal method was the analog, force appropriation method [1-3]. Following that early work, there has been a continuing interest in autonomous methods that, in most cases, have been procedures that are formulated based upon a specific modal parameter estimation algorithm like the Eigensystem Realization Algorithm (ERA), the Polyreference

Time Domain (PTD) algorithm or more recently the Polyreference Least Squares Complex Frequency (PLSCF) algorithm (which thebasis of the commercial version of the PLSCF, the PolyMAX ® method and the rational fraction polynomial algorithm with Z-domain generalized frequency (RFP-z)) [8]. A relatively complete list of autonomous and semi-autonomous procedures that have been reported can be found in a recent paper [4].

Each of these past procedures have shown some promise but have not yet been widely adopted. In many cases, the procedure focused on a single modal parameter estimation algorithm and did not develop a general procedure. Most of the past procedural methods focused on modal frequency (pole) density but depended on limited modal vector data to identify correlated solutions. Currently, due to increased computational speed and availability of memory, procedural methods can be developed that were beyond the computational scope of available hardware only a few years ago. These methods do not require any initial thresholding of the solution sets and rely upon correlation of the vector space of hundreds or thousands of potential solutions as the primary identification tool.

The discussion in the following Sections of the use of multiple modal parameter estimation algorithms in autonomous modal parameter estimation is based upon recent implementation and experience with an autonomous modal parameter estimation procedure referred to as the Common Statistical Subspace Autonomous Mode Identification (CSSAMI) method. The strategy of the CSSAMI autonomous method is to use a default set of parameters and thresholds to allow for all possible solutions from a given data set. This strategy allows for some poor estimates to be identified as well as the good estimates. The philosophy of this approach is that it is easier for the user to evaluate and eliminate poor estimates compared to trying to find additional solutions. The reader is directed to a series of previous papers in order to get an overview of the methodology and to view application results for several cases [4-7].

Note that much of the background of the CSSAMI method is based upon the Unified Matrix Polynomial Algorithm (UMPA) [8]. This means that this method can be applied to both low and high order methods with short or long dimension modal (base) vectors. This also means that most commercial algorithms could take advantage of this procedure. Note that high order, matrix coefficient polynomials normally have coefficient matrices of a dimension that is based upon the short dimension of the data matrix, N_S. In these cases, it may be useful to solve for the complete, unscaled or scaled, modal vector of the large dimension, N_L. This will extend the temporal-spatial information in the modal (base) vector so that the vector will be more sensitive to change. This characteristic is what gives the CSSAMI autonomous method a robust ability to distinguish between computational and structural modal parameters.

2.3 Pole Weighted Modal Vectors

The key to estimating the modal parameters utilizing the CSSAMI autonomous procedure is formulating clusters of pole weighted modal vectors, or state vectors, from the estimates of modal parameters that are represented in a consistency diagram. These state vectors are formed from the modal vector estimates found as the consistency diagram is developed. When comparing modal (base) vectors, at either the short or the long dimension, a pole weighted vector can be constructed independent of the original algorithm used to estimate the poles and modal (base) vectors. For a given order k of the pole weighted vector, the modal (base) vector and the associated pole can be used to formulate the pole weighted vector as follows:

$$\{\phi\}_r = \begin{Bmatrix} \lambda_r^k\{\psi\}_r \\ \cdot \\ \cdot \\ \cdot \\ \lambda_r^2\{\psi\}_r \\ \lambda_r^1\{\psi\}_r \\ \lambda_r^0\{\psi\}_r \end{Bmatrix}_r \qquad \{\phi\}_r = \begin{Bmatrix} z_r^k\{\psi\}_r \\ \cdot \\ \cdot \\ \cdot \\ z_r^2\{\psi\}_r \\ z_r^1\{\psi\}_r \\ z_r^0\{\psi\}_r \end{Bmatrix}_r \tag{3}$$

While the above formulation (on the left) is possible, this form would be dominated by the high order terms if actual frequency units are utilized. Generalized frequency concepts (frequency normalization or Z domain mapping) are normally used to minimize this issue by using the Z domain form (z_r) of the complex modal frequency (λ_r) as shown above (on the right). The Z domain form of the complex natural frequency is developed as follows:

$$z_r = e^{\pi*(\lambda_r/\Omega_{max})} \tag{4}$$

$$z_r^m = e^{m*\pi*(\lambda_r/\Omega_{max})} \tag{5}$$

In the above equations, Ω_{max} can be chosen as needed to cause the positive and negative roots to wrap around the unit circle in the Z domain without overlapping (aliasing). Normally, Ω_{max} is taken to be five percent larger than the largest frequency (absolute value of the complex frequency) identified in the roots of the matrix coefficient polynomial.

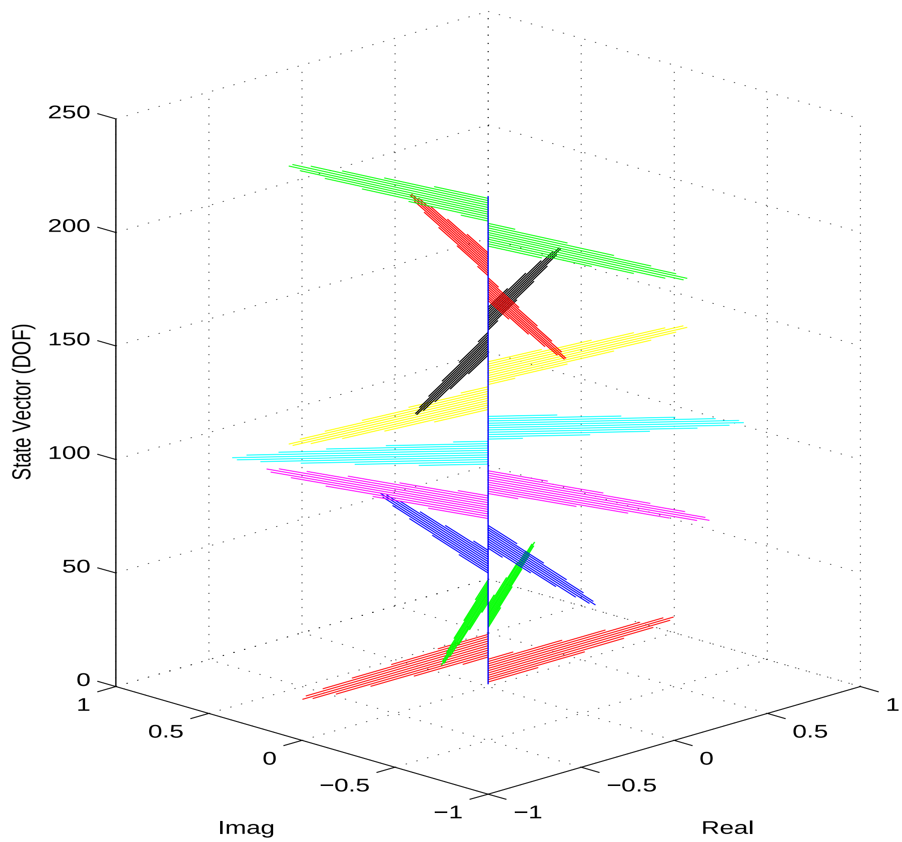

Figure 1. Eighth Order, Pole-Weighted Vector (State Vector) Example

Figures 1 and 2 are graphical representations of the pole weighted vector (state vector) defined in Equation (3). In this example, the modal (base) vector (at the bottom of Figure 1) is a real-valued normal mode that looks like one period of a sine wave. The successive higher orders, up to order eight, are shown in different colors moving up the vertical axis of this figure. The effect of scaling of the modal (base) vector by the higher powers of the Z domain frequency value causes the base vector to rotate in the real and imaginary space. Figure 2 shows the rotation affect clearly. Note that the choice of the order (k) of the pole weighted vector, therefore, just generates additional length and rotation in the pole weighted vector and gives varying sensitivity to comparisons between estimates. Futhermore, note that the choice of order (k) is independent of algorithm. State vectors are a natural part of the numerical formulation for all modal parameter estimation algorithms but this pole weighted vector (state vector), which looks similar, can be formed independently once the modal (base) vector is estimated and, thus, is not constrained by the algorithm. The choice of the order of the pole weighted vector (k) will depend upon the length of the modal (base) vector and is under continuing study at present.

Since the magnitude of the Z domain frequency value is unity, there is no magnitude weighting involved. This rotation gives a method for a single vector to represent the modal (base) vector shape together with the complex-valued frequency. With respect to sorting and separating modal vectors that have similar shapes but different frequencies or similar frequencies but different modal vector shapes, this becomes a powerful parameter, together with modal vector correlation tools like the modal assurance criterion (MAC), for modal parameter estimation and for autonomous modal parameter estimation.

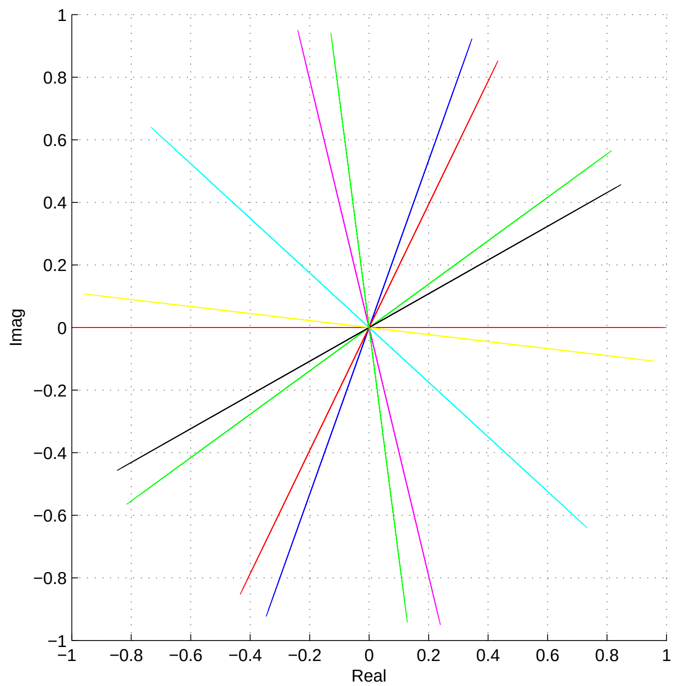

Figure 2. Eighth Order, Pole-Weighted Vector (State Vector) Example - Top View

3. Multi-Algorithm, Extended Consistency Diagrams

Consistency diagrams, historically called stability diagrams, have almost always been utilized and developed for a specific modal parameter estimation algorithm. As such the numerical implementation can be different as a function of basis dimension (N_S or N_L), model order and/or subspace iteration. This would make it very hard to combine different algorithms into a single consistency diagram. However, every algorithm, at the point of the numerical implementation of the consistency diagram, has multiple sets of complex modal frequency and complex-valued modal vectors. The modal vectors may be of different length (N_S or N_L) as a function of algorithm. This potential mismatch in modal (base) vector length can be solved by restricting the long dimension to the DOFs of the short dimension or, more preferably, adding an extra step in the solution procedure to estimate the missing portion of the long dimension vectors, extending them from the short dimension DOFs to the long dimension DOFs. The latter approach is used in the following two figures as an example of *extended consistency diagrams* based upon multiple modal parameter estimation algorithms. In these examples, the results from the individual algorithms are simply stacked into the extended consistency diagram with common sorting and evaluation settings. It should be noted that the order of the stacking of the different algorithms will affect the look of the consistency diagram but the CSSAMI autonomous procedure uses all of the estimated parameters and pays no attention to the sequential ordering and stability calculation involved in the consistency diagram.

The data used for this, and all following examples in this paper, is FRF data taken from an impact test of a steel disc supported in a pseudo free-free boundary condition. The steel disc is approximately 2 cm. thick and 86 cm. in diameter with several small holes through the disc. The center area of the disc (diameter of approximately 25 cm.) has a thickness of approximately 6 cm. There are seven reference accelerometers and measured force inputs from an impact hammer are applied to thirty-six locations, including next to the seven reference accelerometers. The frequency resolution of the data is 5 Hertz. While the disc is not as challenging as some industrial data situations that contain more noise or other complicating factors like small nonlinearities, the disc has a number of pseudo-repeated roots spaced well within the 5 Hertz frequency resolution and a mix of close modes involving repeated and non-repeated roots which are very challenging. Based upon the construction of the disc, real-valued, normal modes can be expected and the inability to resolve these modes can be instructive relative to both modal parameter estimation algorithm and autonomous procedure performance. For the interested reader, a number of realistic examples are shown in other past papers including FRF data from an automotive structure and a bridge structure [4,7].

Figure 3 is an example of using a conventional, sequential sorting procedure involving criteria for frequency, damping and modal vector consistency.

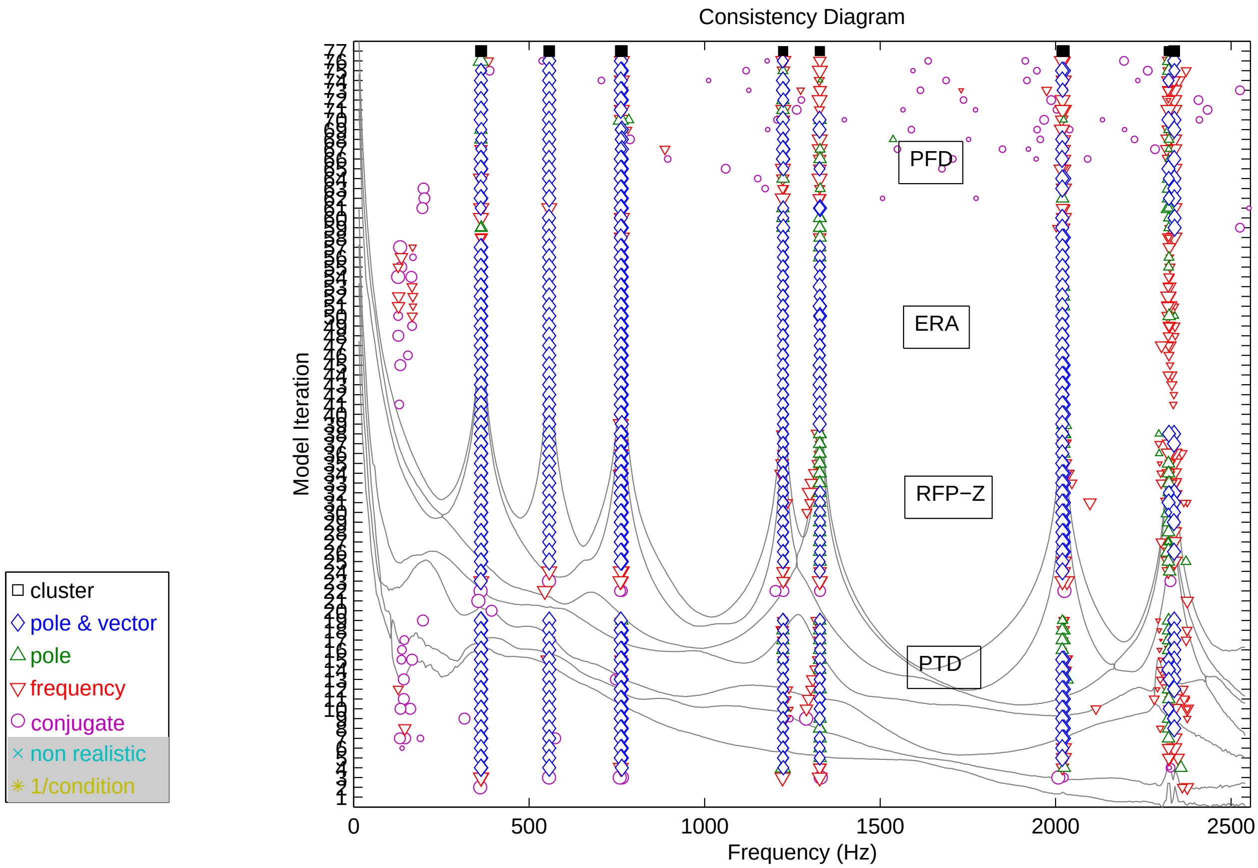

Figure 3. Extended Consistency Diagram - Conventional Version

Figure 4 is an example using a pole weighted vector (state vector) method of producing a similar consistency diagram. In this example, every estimate from every matrix coefficient polynomial solution from every algorithm is converted into a pole weighted vector of a specific order, in this case tenth order. Then, the consistency diagram is developed by using vector correlation methods (MAC) to identify consistency. A similar set of symbols, as those used in Figure 3, are used to define increased levels of consistency as numerical solutions are added.

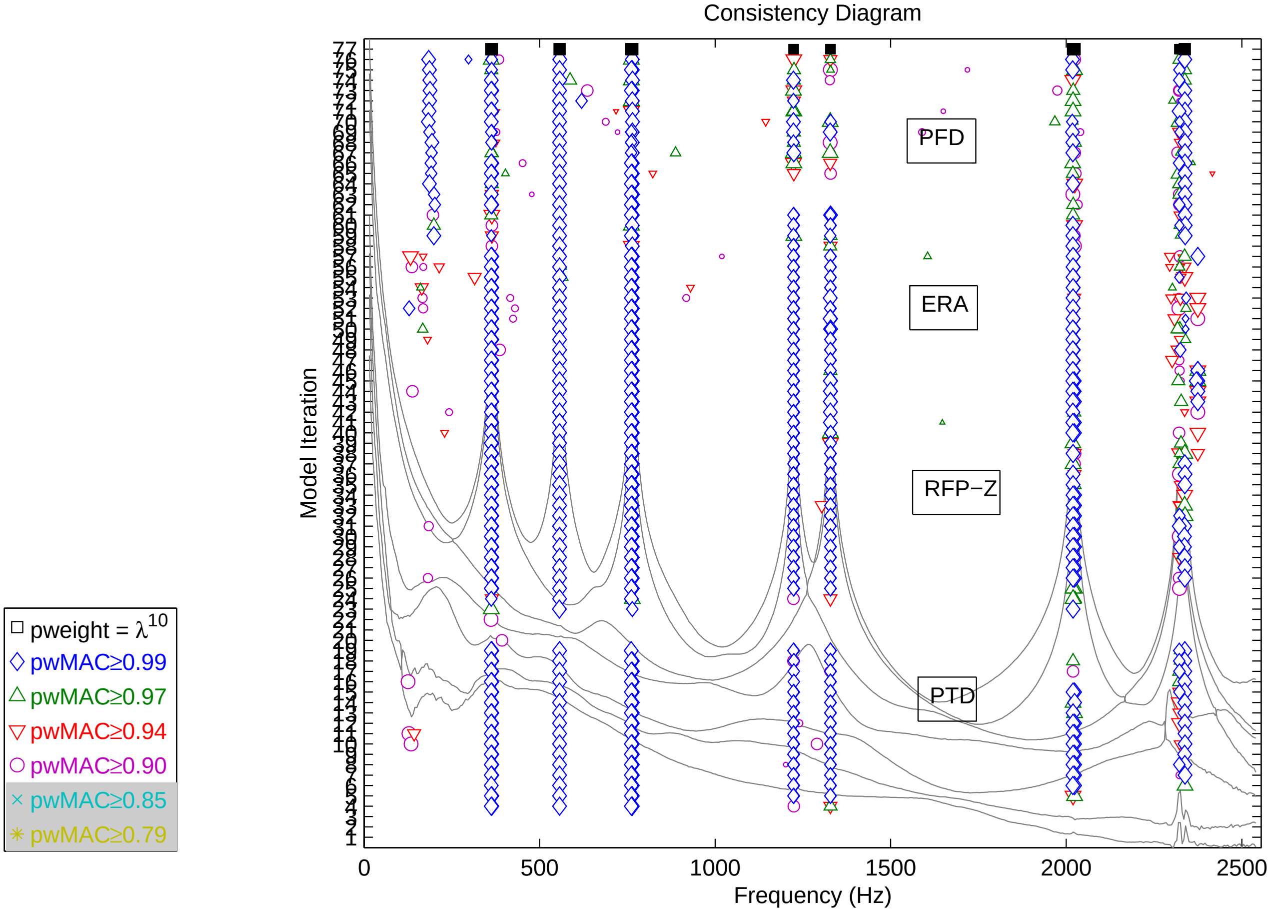

Figure 4. Extended Consistency Diagram - Pole Weighted MAC Version

Both methods work very well but the implementation of Figure 4 is computationally easier and not subject to a frequency drift in the symbol path that can occur in the conventional implementation shown in Figure 1. Note that the solid square symbols at the top of both consistency diagrams represent the solution found from the CSSAMI autonomous modal parameter estimation procedure applied to the information represented by each consistency diagram.

Note that all of the above algorithms are using the same matrix polynomial equation normalization procedure which tends to yield clear consistency diagrams. Each consistency diagram can yield twice as many estimates of the desired modal parameters if both low and high matrix coefficient normalizations are utilized. This is also under current study.

4. Autonomous Modal Parameter Estimation With Extended Consistency Diagrams

The CSSAMI autonomous procedure utilizes all solutions indicated by a symbol in the consistency diagram. If some symbols are not present, it means the user has decided not to view solutions identified by those symbols. This provides a way to remove solutions from the autonomous procedure that are clearly not reasonable. However, experience with the CSSAMI autonomous procedure has shown that some solutions that are often eliminated by users in an attempt to have a clear consistency diagram are often statistically consistent and useful.

Figure 5 shows the solutions that are included in the autonomous procedure. The graphical representation on the left represents a MAC matrix involving the pole weighted vectors for all possible solutions from Figure 3. The graphical representation on the right represents the pole weighted vectors that remain after threshold and cluster size limitations are imposed. Each cluster that remains is evaluated, cluster by cluster, independently to estimate the best modal frequency and modal vector from that cluster. Note that both the positive frequency and negative frequency (complex conjugate) roots are included and identified separately as clusters. Figure 5 represents nearly 1000 solution estimates spanning four different algorithms and 19 different solutions form each algorithm.

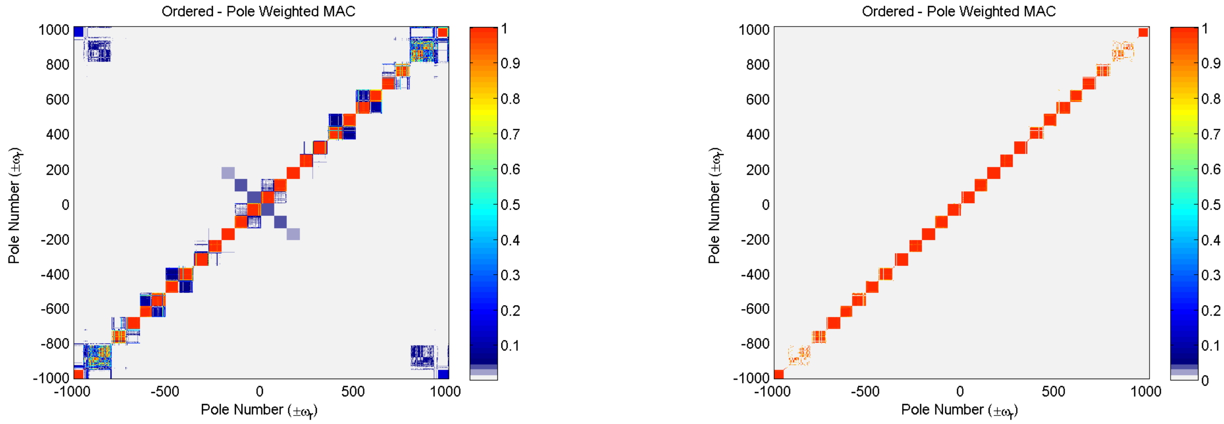

Figure 5. Pole Weighted MAC of All Consistency Diagram Solutions - Before and After Threshold Applied

Once the final set of modal parameters, along with their associated statistics, is obtained, quality can be assessed by many methods that have been used in the past. The most common example is to perform comparisons between the original measurements and measurements synthesized from the modal parameters. Another common example is to look at physical characteristics of the identified parameters such as reasonableness of frequency and damping values, normal mode characteristics in the modal vectors, and appropriate magnitude and phasing in the modal scaling. Other evaluations that may be helpful are unweighted and weighted modal assurance criterion (MAC) evaluation of the independence of the complete modal vector set, mean phase correlation (MPC) of each vector or any other method available. Naturally, since a significant number of pole weighted vectors are used in a cluster to identify the final modal parameters, traditional statistics involving mean and standard deviation are now available.

5. Summary And Future Work

With the advent of more computationally powerful computers and sufficient memory, it has become practical to evaluate sets of solutions involving hundreds or thousands of modal parameter estimates and to extract the common information from those sets. If multiple modal parameter estimation algorithms can be combined into a single autonomous procedure, the statistics related to the common modal parameter estimation become even more meaningful. In most experimental cases studied so far, autonomous procedures give very acceptable results, in some cases superior results, in a fraction of the time required for an experienced user to get the same result.

Future work will involve evaluating alternate numerical methods for combining algorithms into a single consistency diagram (equation normalization, order of the pole weighted vector, etc.) and as well as modal vector solution methods for identifying the best causal results (Do we get a normal mode when we expect a normal mode?). Numerical solution methods that identify both real-valued modal vectors (normal modes) and complex-valued modal vectors, when appropriate, would be truly autonomous.

However, it is important to reiterate that the use of these autonomous procedures or *wizard* tools by users with limited experience is probably not yet appropriate. Such tools are most appropriately used by users with the experience to accurately judge the quality of the parameter solutions identified.

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6. References

[1] Hawkins, F. J., "An Automatic Resonance Testing Technique for Exciting Normal Modes of Vibration of Complex Structures", Symposium IUTAM, "Progres Recents de la Mecanique des Vibrations Lineaires", 1965, pp. 37-41.

[2] Hawkins, F. J., "GRAMPA - An Automatic Technique for Exciting the Principal Modes of Vibration of Complex Structures", Royal Aircraft Establishment, RAE-TR-67-211, 1967.

[3] Taylor, G. A., Gaukroger, D. R., Skingle, C. W., "MAMA - A Semi-Automatic Technique for Exciting the Principal Modes of Vibration of Complex Structures", Aeronautical Research Council, ARC-R/M-3590, 1967, 20 pp.

[4] "Survey of Modal Techniques Applicable to Autonomous/Semi-Autonomous Parameter Identification" ", Allemang, R.J., Brown, D.L., Phillips, A.W.., Proceedings, International Conference on Noise and Vibration Engineering (ISMA), Katholieke Universiteit Leuven, Belgium, 42 pp., 2010.

[5] Phillips, A.W., Allemang, R.J., Brown, D.L., "Autonomous Modal Parameter Estimation: Methodology", Proceedings, International Modal Analysis Conference (IMAC), 22 pp., 2011.

[6] Allemang, R.J., Phillips, A.W., Brown, D.L., "Autonomous Modal Parameter Estimation: Statistical Considerations", Proceedings, International Modal Analysis Conference (IMAC), 17 pp., 2011.

[7] Brown, D.L., Allemang, R.J., Phillips, A.W., "Autonomous Modal Parameter Estimation: Application Examples", Proceedings, International Modal Analysis Conference (IMAC), 26 pp., 2011.

[8] Allemang, R.J., Phillips, A.W., "The Unified Matrix Polynomial Approach to Understanding Modal Parameter Estimation: An Update", Proceedings, International Conference on Noise and Vibration Engineering (ISMA), 2004.