Exciter Impedance and Cross-Axis Sensor Sensitivity Issues in FRF Estimation

Mahmoud Hassan Pour Dargah, Dr. Randall J. Allemang*, Dr. Allyn W. Phillips
School of Dynamic Systems, College of Engineering
University of Cincinnati, 2600 Clifton Ave., Cincinnati, OH 45221-0072

ABSTRACT

This paper studies the effect of shaker impedance and the way this parameter can affect the frequency response measurement (FRF) on structures. Three different shaker designs were used for this study to better understand the effect of the shaker impedance on the force transmissibility characteristic function. Analytical MKC models are used to derive equations and then the experiments were conducted to validate the model. Another effect that is studied here briefly is the cross-sensitivity of the sensors when subjected to lateral forces (or displacements) and bending. It is shown that this effect will lead to non-repeatable measurements, especially in the case of impedance heads.

1. Introduction

The traditional method of making frequency response function (FRF) measurements is to use shakers. The shakers have the capability of applying large forces over wide frequency bands to assure the excitation of vibrational modes of structures. The frequency band that a shaker can sweep and put force on depends on the shaker design and actuation. Electromechanical shakers are capable of putting small to medium forces into a wide frequency band, but as soon as the force requirement goes up, the electrohydraulic ones may become necessary. On the first ones, the magnetic force is being switched electrically to exert force and in the latter, the hydraulic force is being switched electrically. More than one shaker can be used simultaneously to excite a structure, which in turn makes it possible to increase the force level on that structure.

Unfortunately, the shakers must be attached mechanically which can cause numerous problems in accuracy of the collected data. One of the problems associated with using of the shakers is the mass loading of the structure. Having its own mass, stiffness and damping, the change of impedance on the contact point can shift the peaks and change the amplitudes on the estimated FRFs. It has been studied and believed that the smaller the mass of the shaker armature, the less will be the effect of shaker impedance on measurement. It is also worth mentioning that the shaker impedance will affect the amount of force that can be transmitted to the structure. The force transmissibility is an important factor that indicates what portion of the force generated by the shaker has been transferred to the structure. The lower the force, the greater would be the noise to signal ratio that can cause inaccurate measurement. In this study, two designs of pancake shakers have been used in a series of tests in comparison with an industrial grade shaker and the results are demonstrated.

While conducting the tests and analyzing the data, it has been noticed that the type of the transducer used and in general, the design of the specific sensor can also cause inconsistency in the measured data which will be discussed briefly in this paper.

2. Shaker-Structure Interaction/Force Transmissibility

The most important question here is why the force transmissibility is important. Looking back at the definition, the frequency response function is the measure of the structure response to unit force. Having this function will help the designers to get a better idea of how the structure will respond to forcing functions. Ideally when there is no noise, as long as the force and response can be measured through the transducers, the FRF can be estimated easily and accurately. But in the presence of noise, the measured signals must have a high enough level compared to the noise floor in order to get an accurate enough FRF estimation. Some of the FRF estimation methods that are widely used are sensitive to noise and can be biased by its level. It is obvious that the higher the signal to noise ratio, the less these methods are biased and more accurate results can be attained. Care also must be taken as the higher levels of force do not guarantee a better estimation of the FRF as this can raise the non-linearity problems of the structure into estimation. The force transmissibility function can predict what portion of the force will be transmitted to the structure, if it is been amplified or attenuated, and if the phase is leading or lagging.

As mentioned before, attaching the shaker to the structure increases the number of DOFs. One of the major concerns in this set up is the relative magnitude of force that can be transmitted to the structure through the shaker, which is called the force transmissibility function. The force transmissibility indicates that what portion of the force generated by the shaker can be transferred to the structure based on the mechanical characteristics – mass, stiffness, and damping – of shaker and structure and how it will change over the frequency band. A few different cases will be considered to study this effect analytically first.

Case 1: Single degree of freedom system, grounded structure and shaker – Single input single output

In this case, it is assumed that the shaker is connected to the structure directly without using the stinger (Figure 1).

There is one important assumption that is been made here in order to make the calculations easier. It has been assumed that shaker-structure connection (stinger and load cell) is very rigid and there is no relative displacement in between the shaker armature and the point of contact on the structure.

Using the Newton's second law and the equations of motion, the force transmissibility can be expressed in time and frequency domain as below:

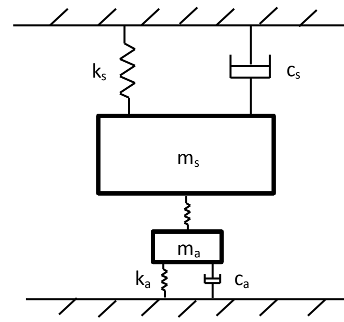

Figure 1 - SODF system excited by one shaker

$$\frac{f}{f_c} = \frac{m_s\ddot{x}+c_s\dot{x}+k_s x}{(m_a+m_s)\ddot{x}+(c_a+c_s)\dot{x}+(k_a+k_s)x} \tag{1}$$

$$\frac{F}{F_c} = \frac{-\omega^2 m_s+j\omega c_s+k_s}{-\omega^2(m_a+m_s)+j\omega(c_a+c_s)+(k_a+k_s)} \tag{2}$$

In which,

m_a: Shaker armature mass
k_a: Shaker stiffness
c_a: Shaker damping
m_s: Structure mass
k_s: Structure stiffness
c_s: Structure Damping
F,f: Transmitted force in frequency and time domain
F_c,f_c: Force generated by shaker in frequency and time domain

As in every transfer function, there exist poles and zeros. By assuming an undamped condition,

$$\omega = \omega_n = \sqrt{\frac{k_s}{m_s}} \tag{3}$$

It can be seen that the natural frequency of the structure is the transfer function zero. This indicates that no matter what type of shaker is used, there will be a force dropout at structure's natural frequency. This phenomenon is also called the force drop-out. And for poles,

$$\omega = \sqrt{\frac{k_s+k_a}{m_s+m_a}} \tag{4}$$

The above equation shows that the pole frequency is a function of both the structure and shaker armature stiffness and damping and, based upon the actual values, can occur either below or above the force dropout frequency.

The equation can be manipulated further for a non-dimensional relationship to explain the dynamics of shaker-structure interaction. By assuming,

$$M = \frac{m_a}{m_s}, \quad K = \frac{k_a}{k_s}, \quad \alpha = \frac{\omega}{\omega_n}$$

The equation turns into:

$$\frac{F}{F_c} = \frac{-\alpha^2+1}{-\alpha^2(M+1)+(K+1)} \tag{5}$$

However, Equation 1 can be viewed another way. By separating the shaker and the structure parameters and using the impedance function instead of equations of motion,

$$\frac{F}{F_c} = \frac{[-\omega^2 m_s+j\omega c_s+k_s]}{[-\omega^2 m_s+j\omega c_s+k_s]+[-\omega^2 m_a+j\omega c_a+k_a]} \tag{6}$$

And by definition,

$$B_s = [-\omega^2 m_s + j\omega c_s + k_s]$$

$$B_a = [-\omega^2 m_a + j\omega c_a + k_a]$$

In which the B(s) are the impedance function of the structure and shaker respectively. By multiplying the numerator and denominator by H_s, the FRF of the structure,

$$\frac{F}{F_c} = \frac{1}{1+B_a H_s} \tag{7}$$

The above equation shows that the force transmissibility is only a function of shakers impedance function and the frequency response function of the structure under test.

Case 2: Multiple degrees of freedom system, grounded structure and shaker – Single input multiple outputs

In this case, Case 1 is extended to a three DOF system with the same assumptions (Figure 2).

Using the Newton's second law and the equations of motion, the force transmissibility can be expressed in the time and frequency domain as below:

$$\frac{f}{f_c} = \frac{m_1\ddot{x}+(c_1+c_2)\dot{x}+(k_1+k_2)x-c_2\dot{x}_2-k_2x_2}{(m_a+m_1)\ddot{x}+(c_a+c_1+c_2)\dot{x}+(k_a+k_1+k_2)x-c_2\dot{x}_2-k_2x_2} \tag{8}$$

$$\frac{F}{F_c} = \frac{[-\omega^2 m_1+j\omega(c_1+c_2)+(k_1+k_2)]X-(j\omega c_2+k_2)X_2}{[-\omega^2(m_a+m_1)+j\omega(c_a+c_1+c_2)+(k_a+k_1+k_2)]X-(j\omega c_2+k_2)X_2} \tag{9}$$

Recalling the H matrix and response-force relationship and the use of a single shaker,

$$\begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix} \begin{Bmatrix} F_1 \\ 0 \end{Bmatrix} = \{X_1 \quad X_2\}$$

$$\frac{X_1}{F} = H_{11}, \quad \frac{X_2}{F} = H_{21}$$

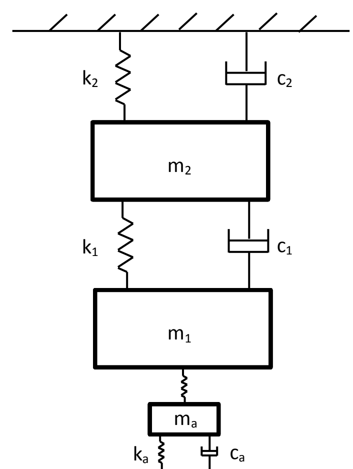

Figure 2 - 2DOF system excited by one shaker

And by substitution, the final equation will be:

$$\frac{F}{F_c} = \frac{[-\omega^2 m_1+j\omega(c_1+c_2)+(k_1+k_2)]H_{11}-(j\omega c_2+k_2)H_{21}}{[-\omega^2(m_a+m_1)+j\omega(c_a+c_1+c_2)+(k_a+k_1+k_2)]H_{11}-(j\omega c_2+k_2)H_{21}} \tag{10}$$

Using the same approach for case one and using matrix operators, the force transmissibility function can also be expressed in impedance function of the shaker and structure under test and its frequency response function. The simplified equation through the same procedure will turn into:

$$\frac{F}{F_c} = \frac{1}{1+B_a H_{11}} \tag{11}$$

As can be seen, when one shaker is used to excite the structure, regardless of structure's degrees of freedom, the force transmissibility is only a function of shaker impedance function and the frequency response function of DOF that the shaker is connected to.

Case 3: Multiple degree of freedom system, grounded structure and shaker – multiple input multiple output

In this case, more than one shaker is used to excite the structure, and shaker/structure interaction will be studied in two separate force transmissibility functions: one for each shaker. All the assumptions are the same as Case 1. The indexes 1 and 2 refer to each shaker that has been used.

Using the Newton's second law and the equations of motion, the force transmissibility can be expressed in time and frequency domain as below. For first shaker,

$$\frac{f_1}{f_{c1}} = \frac{m_1\ddot{x}_1+(c_1+c_2)\dot{x}_1+(k_1+k_2)x_1-c_2\dot{x}_2-k_2x_2}{(m_{a1}+m_1)\ddot{x}_1+(c_{a1}+c_1+c_2)\dot{x}_1+(k_{a1}+k_1+k_2)x_1-c_2\dot{x}_2-k_2x_2} \quad (12)$$

$$\frac{F_1}{F_{c1}} = \frac{[-\omega^2m_1+j\omega(c_1+c_2)+(k_1+k_2)]X_1-(j\omega c_2+k_2)X_2}{[-\omega^2(m_{a1}+m_1)+j\omega(c_{a1}+c_1+c_2)+(k_{a1}+k_1+k_2)]X_1-(j\omega c_2+k_2)X_2} \quad (13)$$

Using the same methodology in Case 2 to further simplify the equations:

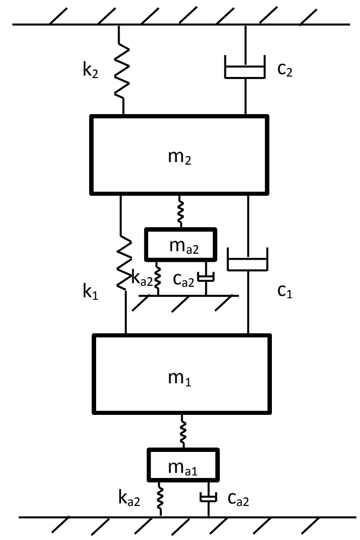

Figure 3 - 2DOF system excited by two shakers

$$\frac{F_1}{F_{c1}} = \frac{1}{1+(-\omega^2m_{a1}+j\omega c_{a1}+k_{a1})\frac{X_1}{F_1}} \quad (14)$$

Or

$$\frac{F_1}{F_{c1}} = \frac{1}{1+B_{a1}\frac{X_1}{F_1}} \quad (15)$$

This time, unfortunately, the equation cannot be simplified anymore. Unlike the Case 2, the presence of the second shaker changes the response calculation and the $H_{11} = \frac{X_1}{F_1}$ equation is not valid for this case.

For the second shaker,

$$\frac{f_2}{f_{c2}} = \frac{m_2\ddot{x}_2+c_2\dot{x}_2+k_2x_2-c_2\dot{x}_1-k_2x_1}{(m_{a2}+m_2)\ddot{x}_2+(c_{a2}+c_2)\dot{x}_2+(k_{a2}+k_2)x_2-c_2\dot{x}_1-k_2x_1} \quad (16)$$

$$\frac{F_2}{F_{c2}} = \frac{[-\omega^2m_2+j\omega c_2+k_2]X_2-(j\omega c_2+k_2)X_1}{[-\omega^2(m_{a2}+m_2)+jw(c_{a2}+c_2)+(k_{a2}+k_2)]X_2-(j\omega c_2+k_2)X_1} \quad (17)$$

Using the same methodology in Case 2 to further simplify the equations:

$$\frac{F_2}{F_{c2}} = \frac{1}{1+(-\omega^2m_{a2}+j\omega c_{a2}+k_{a2})\frac{X_2}{F_2}} \quad (18)$$

Or

$$\frac{F_2}{F_{c2}} = \frac{1}{1+B_{a2}\frac{X_2}{F_2}} \quad (19)$$

There is also another way to represent the force transmissibility function in all the above cases, and that is using the shaker's mechanical impedance instead of its impedance function. By substitution, the force transmissibility function can be expressed using the shaker's mechanical impedance.

For Cases 1 and 2,

$$\frac{F}{F_c} = \frac{1}{1+j\omega Z_a H_{11}} \tag{20}$$

And for Case 3,

$$\frac{F_1}{F_{c1}} = \frac{1}{1+j\omega Z_{a1}\frac{X_1}{F_1}} \quad , \quad \frac{F_2}{F_{c2}} = \frac{1}{1+j\omega Z_{a2}\frac{X_2}{F_2}} \qquad \text{(21 and 22)}$$

As can be seen from the above equations, it can be concluded that the parameters that control the force transmissibility are the frequency, shaker impedance and structure response.

3. Experimental setup

Three different shakers were used to measure the data on the H-frame, two pancake shakers and a PCB/Modal Shop shaker (Figures 4 and 5). The pancakes are referred as core and case, indicating which part of the shaker is the moving part. In pancake-core, the armature is used as the moving part and in pancake-case, the armature casing is used. The armature mass and stiffness of these shakers can be found in Table 1. Force and acceleration were measured on the shaker attachment point on the structure and the FRF was estimated for each case. Shaker bases were hot-glued to the floor prior to each test run in order to minimize the possible movement of the shaker base during the tests. To change the impedance, two measurements were made on each shaker with nylon and metallic stingers. Block size, number of averages, and other parameters used are reported on each plot. The experiments for a single input – single output are presented in this paper and all the other cases will be studied later on.

To measure the force transmissibility, two load cells were used; one on the shaker and one on the structure and the transfer function was estimated through the collected data. Using an accelerometer, the frequency response functions were also estimated and compared for different shakers.

In order to check the repeatability, the measurements were repeated by tearing down and setting up again for two days and for two other days, the measurements were repeated without tearing down the system.

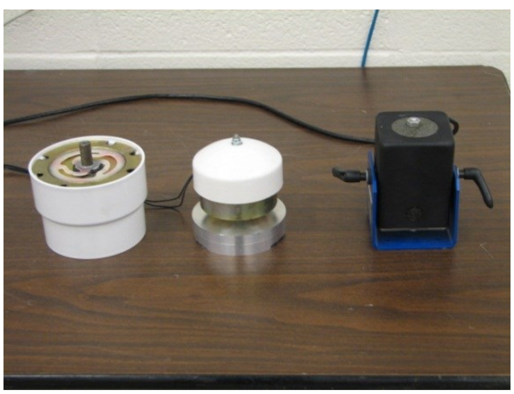

Figure 4 - Shakers from left to right Pancake-Core, Pancake-Case, and PCB 2004E

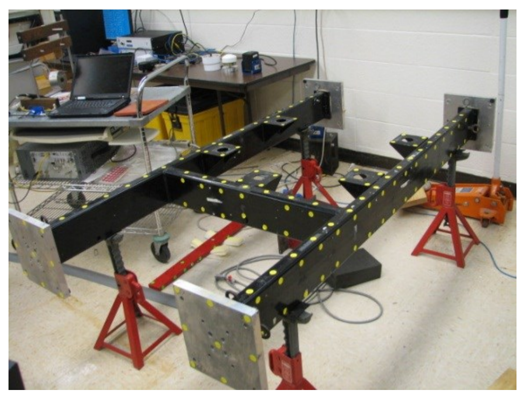

Figure 5 - H-Frame used for experiments

	Pancake-Core	Pancake-Case	PCB – 2004E
Stiffness [N/m]	27.79	30.74	3.48
Total Shaker Mass [gr]	908	1400	2826
Armature Mass [gr]	410	428	32
Base Mass [gr]	498	972	2794
Armature/Base mass [%]	83.84	44.03	1.14

Table 1 - Shakers Mass and Stiffness

4. Results and Discussion

The force transmissibility function measured on the H-frame can be seen in Figure 6. As expected, the pancake shakers with higher impedances were not able to transfer force as the modal shop shaker was. It can also be seen that the resonance of nylon stingers in 500-100 Hz frequency band amplifies the force transferred to the structure due to its drop of impedance. The coherence function drops significantly for the pancake shakers while the modal shop shaker does not have a drop. Passing the 1250Hz, the force transmissibility starts to drop somewhat linearly in all three shakers when the nylon stingers are used. The phase plot also shows that there is that the forcing function also lags the phase in case of nylon stingers. The force drop outs and amplifications are also visible on and around the structure resonance

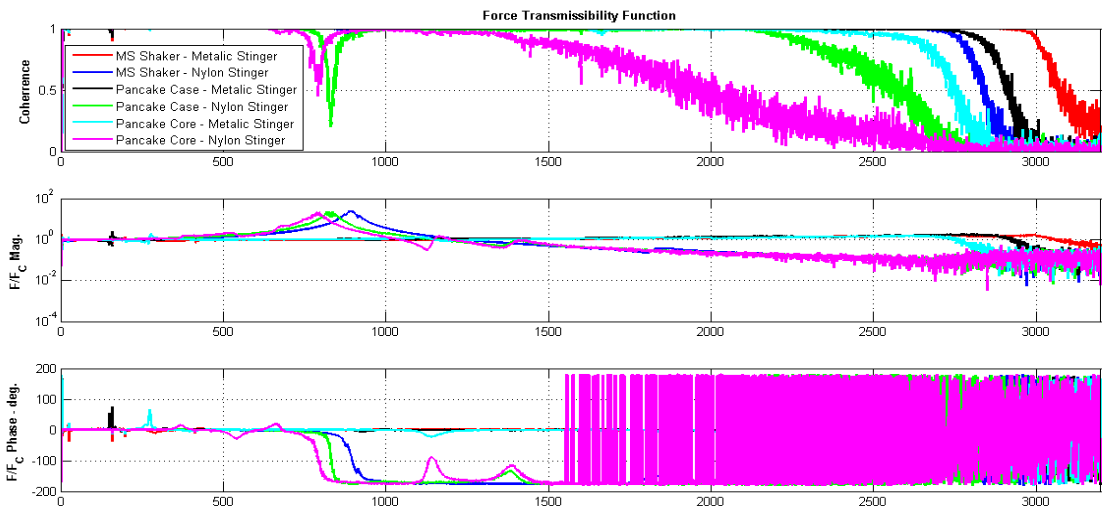

Figure 6 - Force transmissibility function, all shakers, different stinger material (Random input, 8192 Block size, 5 cyclic blocks, 37 averages, Hanning window, H1 estimation, Nyquist frequency 3000Hz)

Not being able to transfer enough force to the structure for excitation, the coherence function on FRF estimation drops significantly due to the lower force levels in pancake shakers, as can be seen in Figure 7. There are also major drops on this function for the modal shop shaker in 1250-1500 frequency bands

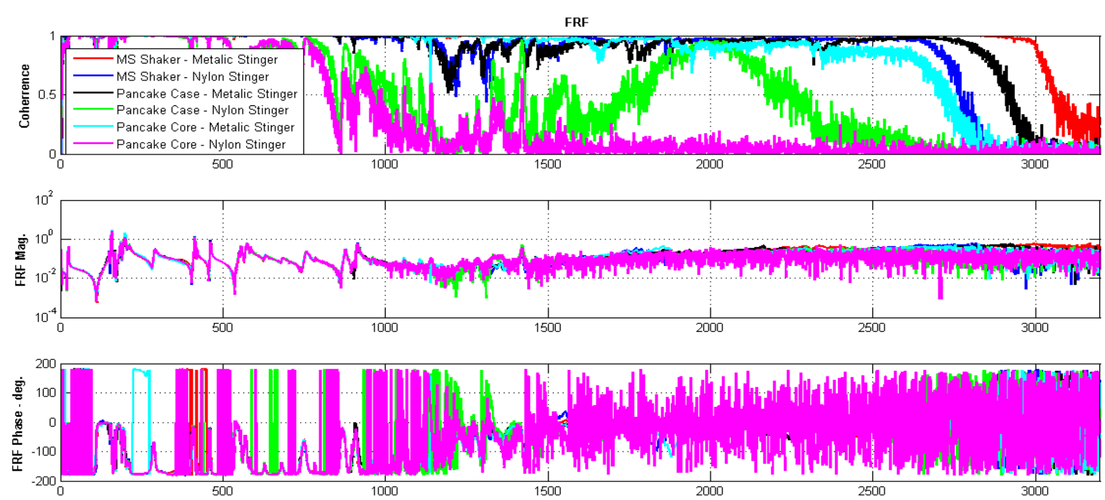

Figure 7 - Frequency Response Function, all shakers, different stinger material, 0-3000Hz (Random input, 8192 Block size, 5 cyclic blocks, 37 averages, Hanning window, H1 estimation, Nyquist frequency 3000Hz)

The results show improvement in case of using the stiffer metallic stingers instead of nylon ones. The force transmissibility function magnitude does not drop before 2500Hz and there is no force amplification or phase lag in any cases. On the frequency response function plot, the coherence function also shows improvement while using metallic stingers instead of nylon ones. In general, it can be seen that the change of impedance of the shaker side changes the amount of force that can be transferred to the structured in a certain setup.

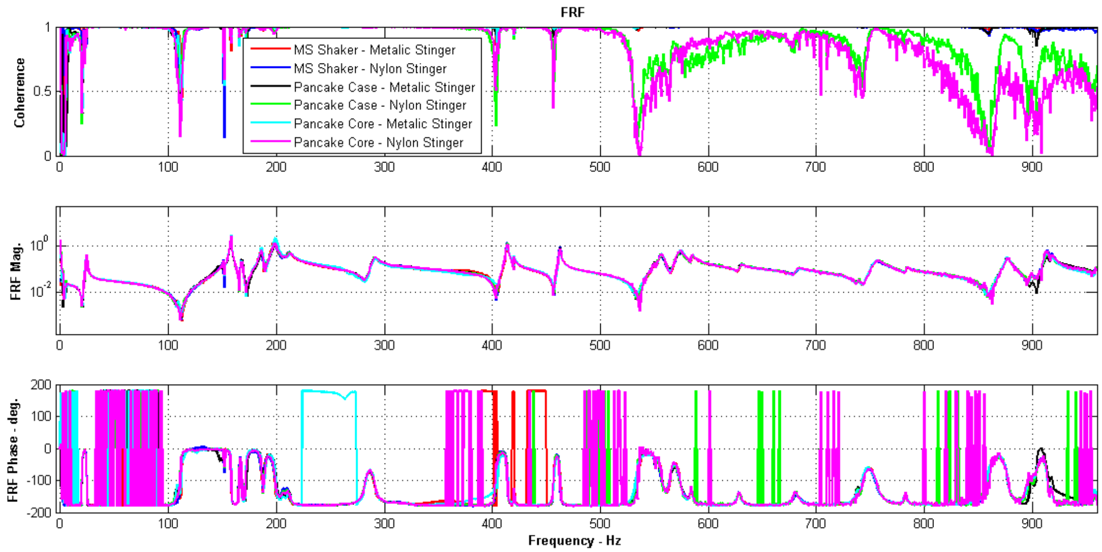

Figure 8 - Frequency Response Function, all shakers, different stinger material , 0-1000Hz (Random input, 8192 Block size, 5 cyclic blocks, 37 averages, Hanning window, H1 estimation, Nyquist frequency 3000Hz)

The other observation that was made during the measurements was the effect of transducers cross-axis sensitivity. It has been noticed that no matter if the system is torn down and reconfigured or just was left over night for next day measurement, there were two frequency bands that the FRF measurements were not repeatable, 170-210 Hz and 380-440 Hz. To check for the possibility of excitation of non-linearities in the system, different force levels were applied but same results were obtained. These results can be seen in Figures 9, 10, and 11.

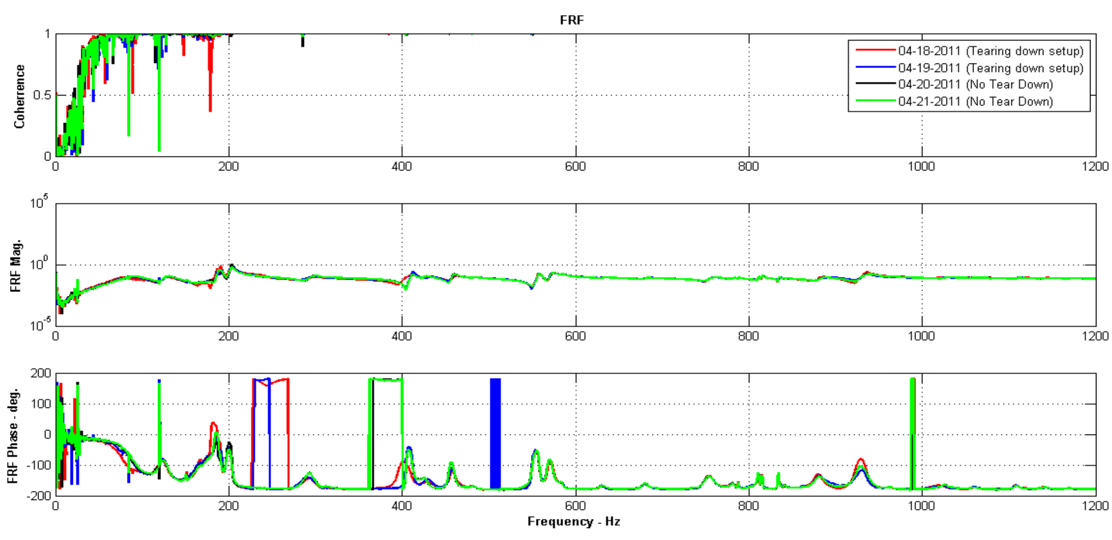

Figure 9 - FRF Repeatability, Modal Shop Shaker, 0-1200 Hz (Random input, 8192 Block size, 5 cyclic blocks, 37 averages, Hanning window, H1 estimation, Nyquist frequency 3000Hz)

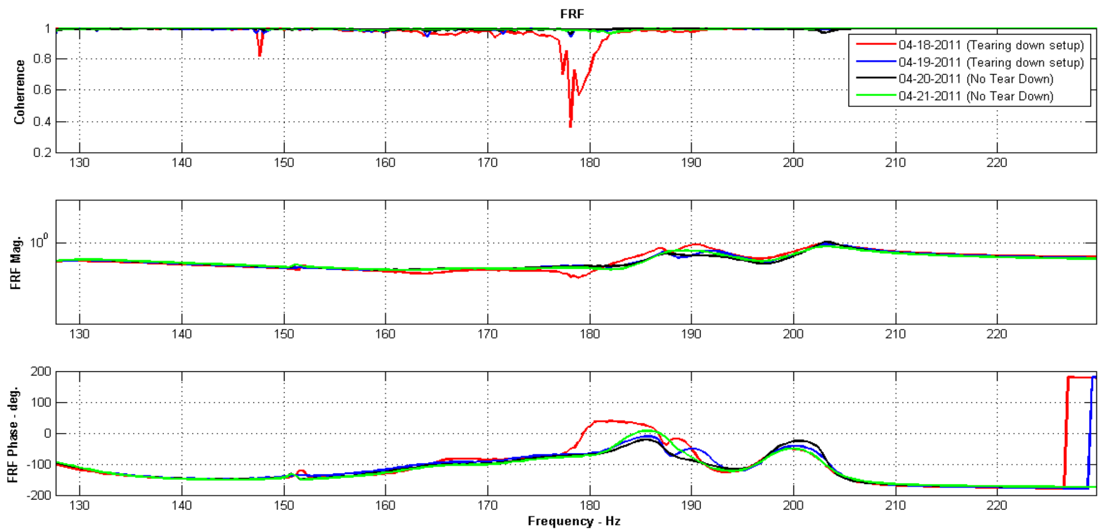

Figure 10 - FRF Repeatability, Modal Shop Shaker, 130-230 Hz (Random input, 8192 Block size, 5 cyclic blocks, 37 averages, Hanning window, H1 estimation, Nyquist frequency 3000Hz)

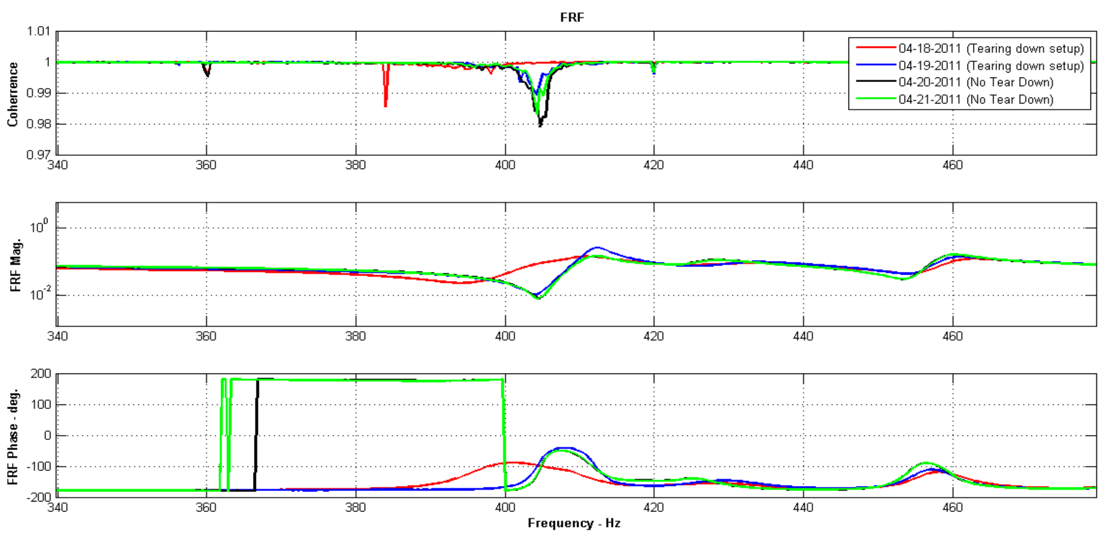

Figure 11 - FRF Repeatability, Modal Shop Shaker, 340-480 Hz (Random input, 8192 Block size, 5 cyclic blocks, 37 averages, Hanning window, H1 estimation, Nyquist frequency 3000Hz)

By using the load cells instead of impedance heads, the repeatability was achieved when the set up was not torn down. By looking at the mode shapes of the structure around those frequencies, it was noticed that the node that the shakers were connected to are subjected to bending. Being subjected to bending and lateral forces-displacement seems to be the main reason for the inconsistency of the results. Unlike the accelerometers, unfortunately there are not many resources available that study the effect of bending on the load cells and accelerometers.

5. Conclusions

Three different cases were studied analytically to better understand the dynamics of shaker-structure interaction through the force transmissibility characteristic function. It has been shown that depending on the set up, the amount of force that can be transferred to the structure is a function of frequency, shaker impedance and the structure response on that specific frequency. The stinger material can also affect the force transmissibility function as it changes the impedance on the shaker side.

The placement of the load cell and accelerometers on the structure can lead to measurement inconsistencies due to the cross-axis sensitivity of the sensors. It was also observed that impedance heads are more sensitive in these cases.

References