Autonomous Modal Parameter Estimation: Statisical Considerations

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ABSTRACT

Autonomous modal parameter estimations may involve sorting a large number of possible solutions to develop one consistent estimate of the modal parameters (frequency, damping, mode shape, and modal scaling). Once the final, consistent estimate of modal parameters is established, this estimate can be compared to related solutions from the larger set of solutions to develop statistical attributes for the final, consistent set of modal parameters. These attributes will include sample size, standard deviation and other familiar variance estimates. New variance estimates are introduced to categorize the modal vector solution. These modal vector statistics are based upon the residual contributions in a set of correlated modal vectors that are used to estimate a single modal vector. Examples of this statistical information is included for a number of realistic data cases.

Nomenclature

N = Number of vectors in cluster.
σ_r = Singular value r from cluster.
λ_r = S domain polynomial root.
λ_r = Complex modal frequency (rad/sec).
z_r = Z domain polynomial root.
$\{\psi_r\}$ = Base vector (modal vector).
$\{\phi_r\}$ = Pole weighted base vector (state vector).
Std. Dev. = Standard deviation.
$NMVR_1$ = Normalized modal vector residual 1.
$NMVR_2$ = Normalized modal vector residual 2.
$NSVR_1$ = Normalized state vector residual 1.
$NSVR_2$ = Normalized state vector residual 2.

1. Introduction

The desire to estimate modal parameters automatically, once a set or multiple sets of test data are acquired, has been a subject of great interest for more than 40 years [1-24]. In the 1960s, even when modal testing was limited to analog test methods, several researchers were exploring the idea of an automated test procedure for determining modal parameters [1-3]. Today, with the increased memory and compute power of current computers used to process test data, an automated or autonomous, modal parameter estimation procedure is entirely possible and is being attempted by numerous researchers.

During the development of a new autonomous modal parameter estimation procedure, it became obvious that, since a large number of possible solutions were being evaluated, that this development was a natural way to introduce statistical evaluations into the modal analysis estimation process. This paper reviews some of the statistical estimates that can aid any user in evaluating possible modal parameter estimation solutions.

The larger question concerning autonomous modal parameter estimation is the intended user. Is the autonomous modal parameter estimation procedure expected to give results sufficiently robust for the novice user? This implies that the user could have no experience with modal analysis and, therefore, have no experiential judgement to use in assessing the quality of the results. The use of the term *wizard* implies that this is the desired situation. In contrast, the user could be very knowledgeable in the theory and experienced. For this case, the autonomous modal parameter estimation procedure is simply an efficient mechanism for sorting a very large number of solutions into a final set of solutions that satisfies a set of criteria

and thresholds that are acceptable to the user. This user is the assumed reader for the purposes of this discussion. The statistical parameters introduced in this paper, however, are designed to give all users additional information that will allow both experienced and novice users to successfully identify the modal parameters, within the limits of the information provided by the measured data.

In order to discuss autonomous modal parameter estimation, some background is needed to clarify terminology and methodology. The reader is directed to a series of companion papers in order to get an overview of the methodology and to view application results for several cases [25-26]. The statistics explained in this paper are part of this new general procedure for autonomous modal parameter that is based upon consistent state vectors. This method is referred to as **Common Statistical Subspace Autonomous Mode Identification (CSSAMI).** Note that much of the background of the CSSAMI method is based upon the Unified Matrix Polynomial Algorithm (UMPA) developed by the authors and described in a number of other papers [27-29].

2. Background: Autonomous Modal Parameter Estimation

The interest in automatic modal parameter estimation methods has been documented in the literature since at least the mid 1960s when the primary modal method was the analog, force appropriation method [1-3]. Following that early work, there has been a continuing interest in autonomous methods [4-24] that, in most cases, have been procedures that are formulated based upon a specific modal parameter estimation algorithm like the Eigensystem Realization Algorithm (ERA), the Polyreference Time Domain (PTD) algorithm or more recently the Polyreference Least Squares Complex Frequency (PLSCF) algorithm or the commercial version of the PLSCF, the PolyMAX ® method.

Each of these past procedures have shown some promise but have not yet been widely adopted. In many cases, the procedure focussed on a single modal parameter estimation algorithm and did not develop a general procedure. Most of the past procedural methods focussed on pole density but depended on limited modal vector data to identify correlated solutions. Currently, due to increased computational speed and larger availability of memory, procedural methods can be developed that were beyond the computational scope of available hardware only a few years ago. These methods do not require any initial thresholding of the solution sets and rely upon correlation of the vector space of thousands of potential solutions as the primary identification tool. With the addition to any modal parameter estimation algorithm of the concept of pole weighted base vector, the length, and therefore sensitivity, of the extended vectors provides an additional tool that appears to be very useful.

3. Autonomous Modal Parameter Estimation Method

The autonomous modal parameter estimation method developed and presented in the following is a general method that can be used with any algorithm that fits within the UMPA structure. A complete description of the Unified Matrix Polynomial Algorithm (UMPA) thought process can be found in a number of references developed by the authors [27-29]. This means that this method can be applied to both low and high order methods with low or high order base vectors. This also means that most commercial algorithms could take advantage of this procedure. Note that high order matrix coefficient polynomials normally have coefficient matrices of dimension that is based upon the short dimension of the data matrix ($N_S \times N_S$). In these cases, it may be useful to solve for the complete modal vector in addition to using the extended base vector as this will extend the temporal-spatial information in the base vector so that the vector will be more sensitive to change. This characteristic is what gives this autonomous method the ability to distinguish between computational and structural modal parameters.

The implementation of the autonomous modal parameter estimation for this method is briefly outlined in the following steps. For complete details, please see the associated papers [25-26].

- Develop a consistency diagram using any UMPA solution method. Since this autonomous method utilizes a pole surface density plot, having a large number of iterations in the consistency diagram (due to model order, subspace iteration, starting times, equation normalization, etc.) will be potentially advantageous. However, the larger the number of solutions (represented by symbols) in the consistency diagram, the more computation time and memory will be required. However, restricting the number of solutions using clear stabilization (consistency) methods may be counterproductive.
- If the UMPA method is high order (coefficient matrices of size $N_S \times N_S$), solve for the complete length, scaled vector (function of N_L for all roots, structural and computational).

- Based upon the pole surface density threshold, identify all possible pole densities above some minimum value. This will be a function of the number of possible solutions represented by the consistency diagram.
- Sort the remaining solutions into frequency order based upon damped natural frequency (ω_r).
- Construct the 10th order, pole weighted vector (state vector) for each solution.
- Normalize all pole weighted vectors to unity length with dominant real part.
- Calculate the Auto-MAC matrix for all pole weighted vectors.
- Retain all Auto-MAC values that have a pole weighted MAC value above a threshold, 0.8 works well for most cases. All values below the threshold are set to 0.0.
- Identify vector clusters from this pole weighted MAC diagram that represent the same pole weighted vector. This is done by a singular value decomposition (SVD) of the pole weighted MAC matrix. The number of significant singular values for this MAC matrix represents the number of significant pole clusters in the pole weighted vector matrix and the value of each significant singular value represents the size of the cluster since the vectors are unitary. Note that the singular value is nominally the square of the number of vectors in the cluster and will likely be different, mode by mode.
- For each significant singular value, the location of the corresponding pole weighted vectors in the pole weighted vector matrix (index) is found from the associated left singular vector. This is accomplished by multiplying the left singular vector by the square root of the singular value and retaining all positions (indexes) above a threshold (typically 0.9). The positions of the non-zero elements in this vector are the indexes into the pole weighted vector matrix for all vectors belonging to a single cluster.
- For each identified pole cluster, perform a singular value decomposition (SVD) on the set of pole weighted vectors. The significant left singular vector is the dominant (average) pole weighted vector. Use the zeroth order portion of this dominant vector to identify the modal vector and the relationship between the zeroth order and the first order portions of the dominant vector to identify the modal frequency and modal damping values.
- Estimate appropriate statistics for each mode identified based upon the modes that are grouped in each cluster.
- For the modal parameters identified, complete the solution for modal scaling using any MIMO process of your choice.
- User interaction with the final set of values can exclude poorly identified modes based upon physical or statisical evaluations.

Once the final set of modal parameters, along with their associated statistics, is obtained, quality can be assessed by many methods that are currently available. The most common example is to perform comparisons between the original measurements and measurements synthesized from the modal parameters. Another common example is to look at physical characteristics of the identified parameters such as reasonableness of frequency and damping values, normal mode characteristics in the modal vectors, and appropriate magnitude and phasing in the modal scaling. Other evaluations that may be helpful are mean phase correlation (MPC) on the vectors, an Auto-MAC looking for agreement between the modal vectors from conjugate poles or any other method available.

4. Statistical Evaluation Parameters

Statistical evaluation parameters can be estimated for each common cluster of pole weighted modal vectors on the basis of the complex modal frequency, the modal vector, and the modal scaling. The number of pole weighted vectors will in general be different in each cluster so the statistics will be based upon the number of estimates available (sample size N). Examples of the statistics currently computed for each modal parameter are described in the following sections.

4.1 Modal Frequency Statistics

The weighted modal frequency for the cluster is found by constructing the pole weighted vector (typically 10th order) for each pole retained in a cluster, then taking the SVD of the group of pole weighted vectors and selecting the singular vector associated with the largest singular value. This chosen singular vector contains both the shape and the modal frequency information. The modal frequency is identified by dividing the first order portion by the zeroth order portion of the vector in a least squares sense. (Note that it is also possible to solve the frequency polynomial which would result from using the complete vector.) Also, for numerical reasons, the pole weighted vector is actually computed in the Z-domain.

$$\{\phi\}_r = \begin{Bmatrix} z_r^v\{\psi\}_r \\ \cdot \\ \cdot \\ \cdot \\ z_r^2\{\psi\}_r \\ z_r^1\{\psi\}_r \\ z_r^0\{\psi\}_r \end{Bmatrix}_r$$

For comparison purposes, the actual mean value and standard deviation of the poles (as well as, the separate frequency and damping means and standard deviations) which were used in the computation of the weighted solution are computed. Since these results are unweighted by the vector characteristics, they will be somewhat different from the vector weighted solution and provide comparitive feedback about the pole.

4.2 Modal Vector Statistics

In order to evaluate the quality of the resulting modal vectors, several different parameters (representing noise to signal ratios) are calculated. These ratios are evaluated for both the original normalized vectors and the pole weighted (state extended) vectors and are computed using the singular value decomposition of each of the set of vectors.

4.2.1 Normalized Modal Vector Residual (NMVR)

The first modal vector parameter is evaluated by taking the total residual magnitude (the Forbenius norm of the residuals) divided by the magnitude of the principal vector magnitude. In other words, the square root of the sum of the squares of the residual singular values divide by the first (largest) singular value. This provides an indication of the consistency of the original contributing vectors. Small values tend to indicate greater consistency. Large values indicate greater variance or the possibility that more than one mode has been included in a cluster.

$$\lceil \sigma \rfloor = \mathrm{SVD}\left(\left[\psi_1 \psi_2 \cdots \psi_N\right]\right) \tag{1}$$

$$\mathrm{NMVR}_1 = \frac{\sqrt{\dfrac{\sum_{k=2}^{N} \sigma_k^2}{N}}}{\sigma_1} \tag{2}$$

The second modal vector parameter is evaluated by taking the largest residual magnitude divided by the magnitude of the principal vector magnitude. In other words, the second singular value divided by the first singular value. This provides an indication of the consistency of the original contributing vectors. A small value tends to indicate random variance. A large value can indicate a consistent modal contamination of the original vectors, possibly caused by a second mode included in the cluster.

$$\mathrm{NMVR}_2 = \frac{\sigma_2}{\sigma_1} \tag{3}$$

4.2.2 Normalized State Vector Residual (NSVR)

The associated state vector parameters are calculated analogous to the above except that the complete pole weighted (state extended) vector is used.

$$\lceil \sigma \rfloor = \mathrm{SVD}\left(\left[\phi_1 \phi_2 \cdots \phi_N\right]\right) \tag{4}$$

$$\mathrm{NSVR}_1 = \frac{\sqrt{\dfrac{\sum_{k=2}^{N} \sigma_k^2}{N}}}{\sigma_1} \tag{5}$$

$$\mathrm{NSVR}_2 = \frac{\sigma_2}{\sigma_1} \tag{6}$$

4.3 Modal Scaling Statistics

If modal scaling (residue) information is available in the original vectors, it can be applied to the principal pole weighted vector such that the consistency of modal scaling (Modal A) can also be evaluated. By scaling the largest driving point response to unity, a uniform, comparable value for Modal A is chosen. Since the modal scaling parameter (Modal A) contains the physical units and hence the relative contribution of each pole/vector combination to the original data set, a means of evaluating the significance and confidence in each estimate is possible. Because the original vectors represent multiple scaled solutions and since Modal A is a single scalar complex value for each pole/vector combination, the statistical distribution of Modal A (mean and standard deviation) of the original cluster can be compared with the scaling estimated from the pole weighted (state extended) vector. The magnitude of Modal A can be used to evaluate the contribution of this vector to the total set of identified vectors. When the user believes that the identified vectors should be normal modes, the phase of the Modal A can be used to evaluate the closeness to a normal mode.

5. Typical Statistics: C-Plate Example

For this example, impact data from a circular plate test article has been used. The FRF data (7x36) was processed using a Rational Fraction Polynomial algorithm with complex z frequency mapping (RFP-Z) with all data between 220 and 2450 Hz included. The model order range used was 2 to 20 with a generalized residual model including all terms from $(j\omega)^{-4}$ to $(j\omega)^{2}$. Further, low order alpha coefficient normalization has been used and full length phase one residues have been calculated.

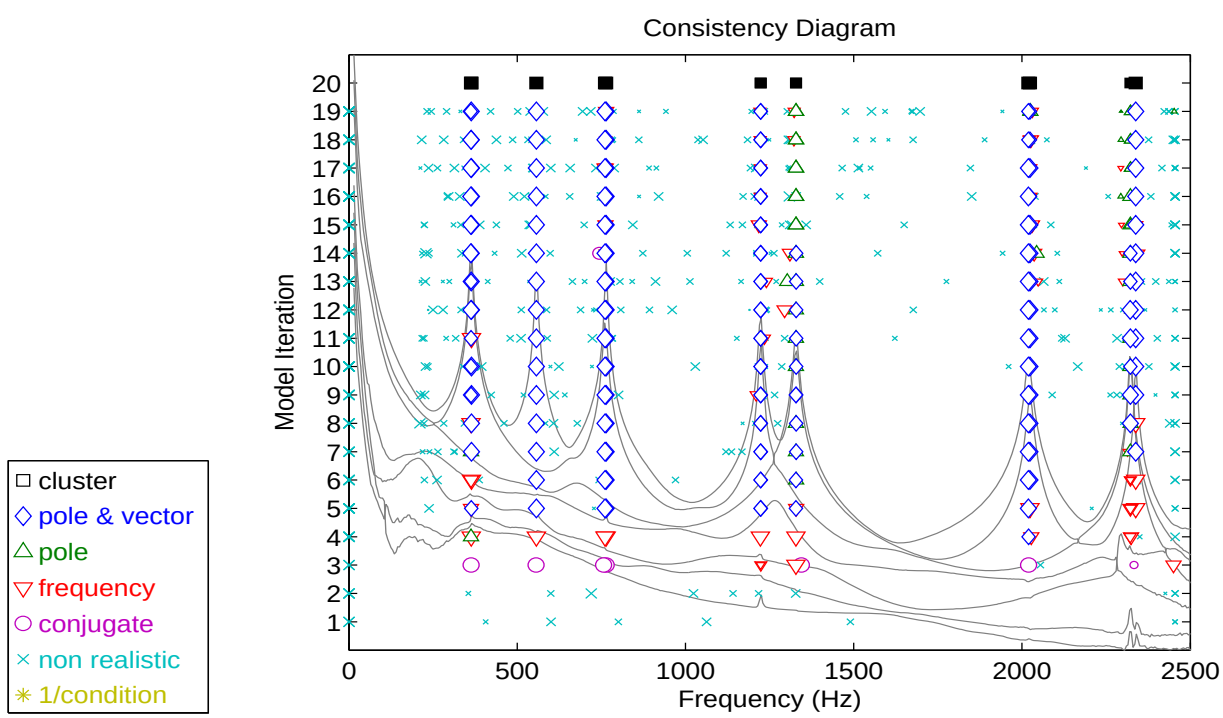

Figure 1. C-Plate Example: Consistency Diagram - All Poles Plotted

For the purposes of demonstration, all calculated poles (both consistent and computational) are included in the plots. As can be observed, though clearly present throughout the frequency range (Figure 1), the non-realistic, computational poles, are not observed in the vicinity of the S-domain computed pole clusters (Figures 2, 3 and 4.)

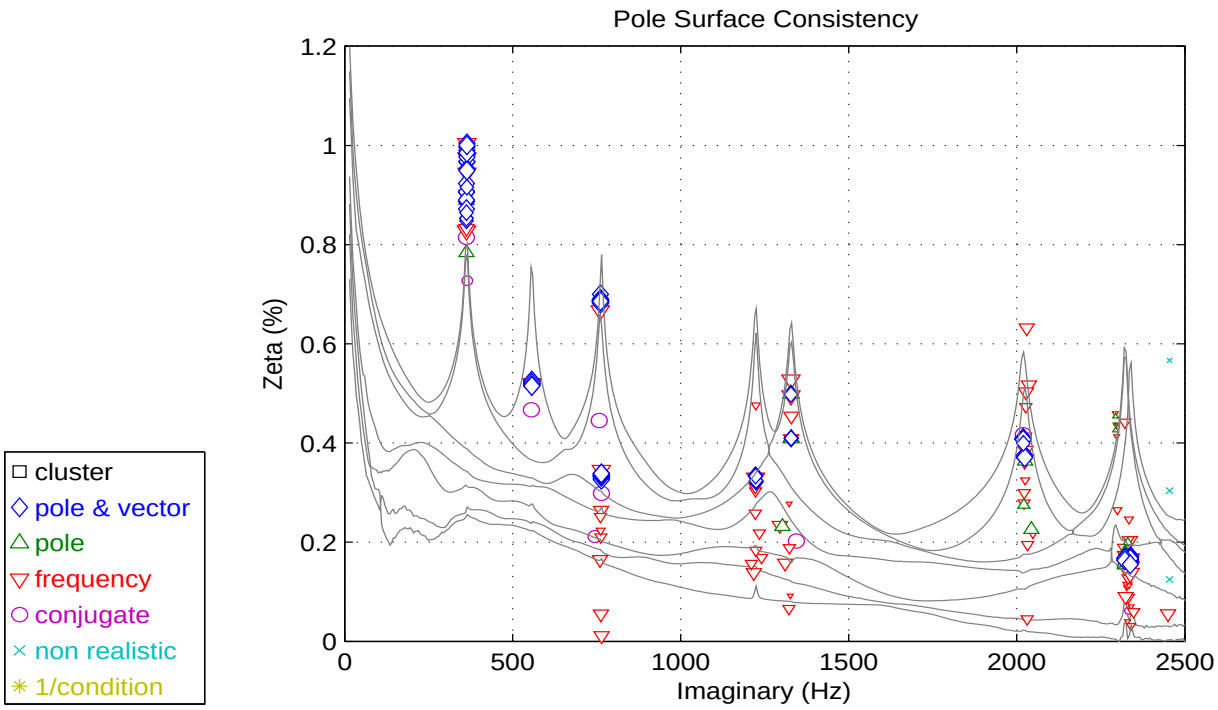

Figure 2. C-Plate Example: Pole Surface Consistency - All Poles Plotted

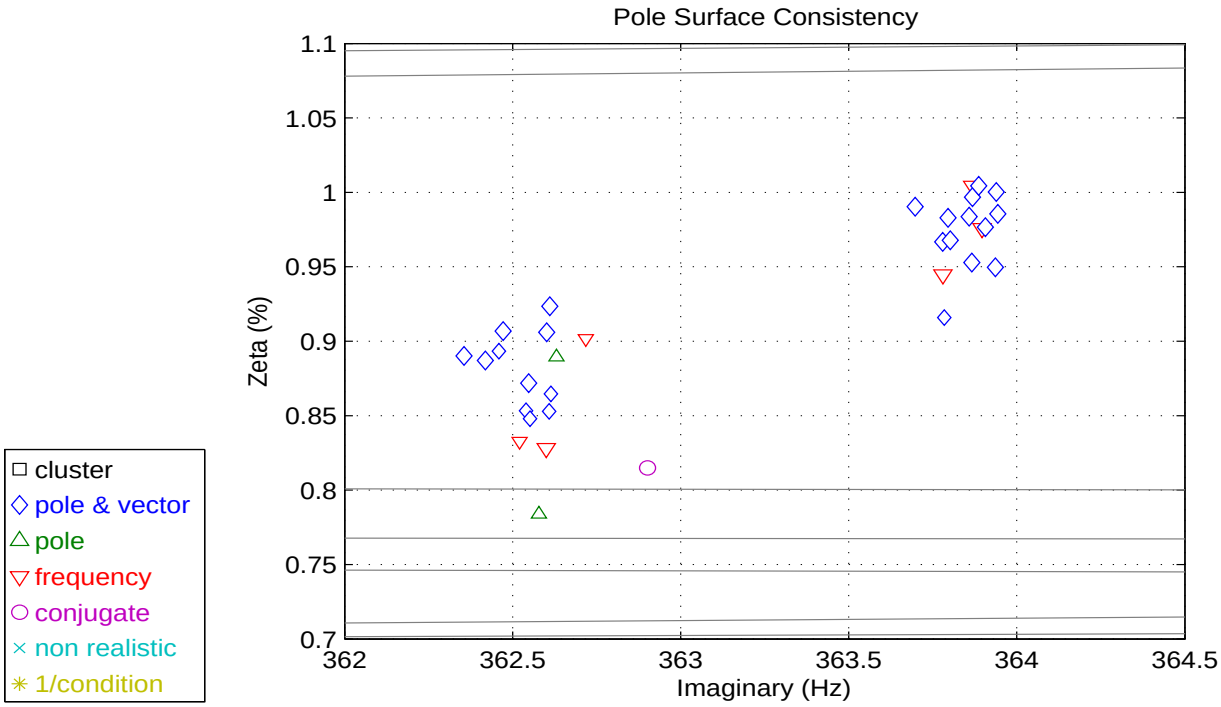

Figure 3. C-Plate Example: Zoomed Pole Clusters - Damping %ζ

The following plots, Figures 5 and 6, are a presentation of the statistical distribution of the pole information for the two clusters (pseudo-repeated root) shown in Figure 4. In Figures 5 and 6, the black stars are the original pole estimates and the red star represents the mean pole calculated from this cluster of pole estimates. The large red circle is the one standard deviation radius. The green square is the indvidual standard deviations for the frequency and the damping. The blue circle is the pole as estimated from the pole weighted (state extended) vector.

Note that the scaling for the two different mode clusters in Figure 5 is consistent so that larger diameter circles will represent more variability.

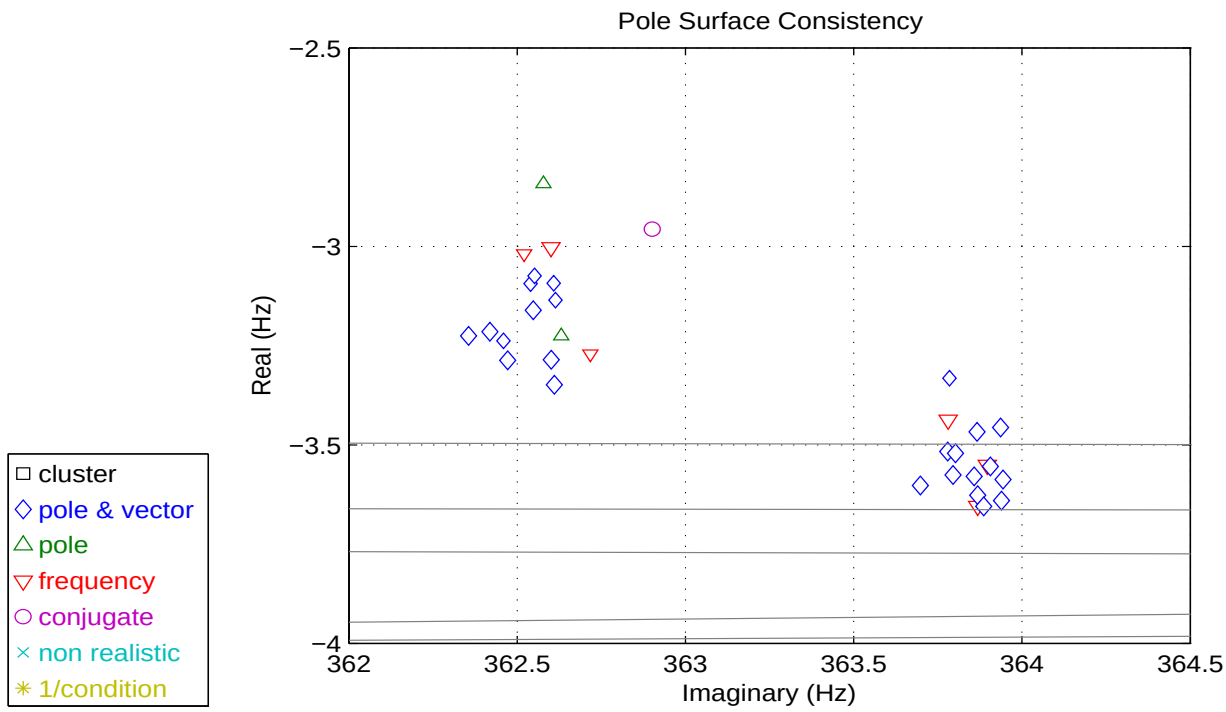

Figure 4. C-Plate Example: Zoomed Pole Clusters - Damping Hz

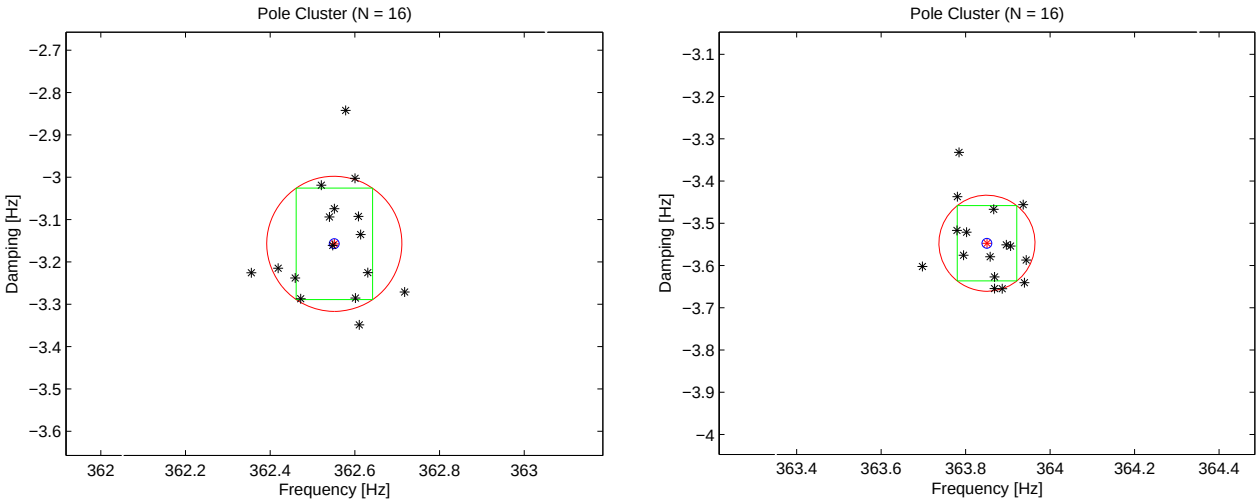

Figure 5. C-Plate Example: 362.6 Hz and 363.9 Hz Pole Clusters - Statistics

The following (Tables 1, 2, and 3) provide a summary of the statistical information evaluated on the C-Plate test structure.

Note that in Table 1 that there is little difference in the complex modal frequency estimated from the most significant pole weighted vector (columns 3 and 4) when compared to the mean values of the complex portions of the modal frequency (columns 5 and 6).

Index	Sample Size (N)	Frequency (Hz)	Damping (Hz)	Frequency (Hz) (Mean)	Damping (Hz) (Mean)	Std. Dev. (Hz)
8	16	362.564	-3.1666	362.564	-3.1670	0.1677
19	16	363.860	-3.5650	363.860	-3.5650	0.1251
10	16	557.055	-2.8966	557.055	-2.8966	0.0795
12	16	761.224	-5.2229	761.224	-5.2229	0.1008
2	17	764.190	-2.5371	764.190	-2.5371	0.1529
16	16	1222.980	-4.0883	1222.980	-4.0883	0.0271
13	16	1224.055	-3.9540	1224.055	-3.9540	0.0419
4	17	1328.036	-6.6495	1328.036	-6.6507	0.0907
17	16	1328.803	-5.4761	1328.803	-5.4768	0.1871
6	17	2019.269	-8.2512	2019.271	-8.2511	0.2543
23	15	2023.802	-7.5597	2023.801	-7.5598	0.0642
26	10	2321.862	-3.9074	2321.860	-3.9070	0.1690
28	8	2322.335	-3.8356	2322.335	-3.8367	0.2244
22	15	2337.895	-3.7967	2337.895	-3.7971	0.2287

TABLE 1. C-Plate Example: Modal Frequency Statistics

In Table 2, note that the fraction of normalized residual (columns 4-7) are all quite small indicated that only one mode has been found in each cluster and that the modal vectors in the cluster were essentially the same modal vector with minimal differences (noise).

Index	N	Frequency (Hz)	$NMVR_1$	$NMVR_2$	$NSVR_1$	$NSVR_2$
8	16	362.564	0.0046	0.0730	0.0046	0.0730
19	16	363.860	0.0034	0.0543	0.0034	0.0543
10	16	557.055	0.0001	0.0014	0.0001	0.0014
12	16	761.224	0.0007	0.0092	0.0007	0.0093
2	17	764.190	0.0020	0.0312	0.0020	0.0312
16	16	1222.980	0.0017	0.0213	0.0017	0.0213
13	16	1224.055	0.0013	0.0185	0.0013	0.0184
4	17	1328.036	0.0042	0.0718	0.0042	0.0717
17	16	1328.803	0.0022	0.0342	0.0022	0.0342
6	17	2019.269	0.0054	0.0763	0.0054	0.0763
23	15	2023.802	0.0047	0.0537	0.0047	0.0537
26	10	2321.862	0.0162	0.1617	0.0162	0.1617
28	8	2322.335	0.0161	0.1279	0.0161	0.1278
22	15	2337.895	0.0020	0.0238	0.0020	0.0238

TABLE 2. C-Plate Example: Modal Vector Statistics

In Table 3, note that the method used to compute the modal scaling, Modal A, had little effect on the estimates of Modal A. Also of note in this case, is the phase angle of all Modal A terms is around minus ninety degrees. The sign on the phase angle and the closeness to ninety degrees (rather than zero degrees) is a function of the scaling method chosen for the modal vector.

Index	N	Frequency (Hz)	ModalA (State Vector)				ModalA (Mean from cluster)				
			real	imaginary	magnitude	phase	real	imaginary	magnitude	phase	Std. Dev.
8	16	362.564	3.9663e+002	-5.7201e+003	5.7338e+003	-86.03	3.9574e+002	-5.7195e+003	5.7332e+003	-86.04	1.3866e+002
19	16	363.860	-4.8353e+001	-4.8716e+003	4.8719e+003	-90.57	4.5311e+001	-4.9271e+003	4.9273e+003	-89.47	3.1271e+002
10	16	557.055	7.3166e+001	-9.7205e+003	9.7207e+003	-89.57	8.4818e+001	-9.7107e+003	9.7111e+003	-89.50	1.2295e+002
12	16	761.224	9.6911e+001	-1.8433e+004	1.8434e+004	-89.70	9.5750e+001	-1.8433e+004	1.8433e+004	-89.70	2.1414e+002
2	17	764.190	-3.1250e+002	-7.0511e+003	7.0580e+003	-92.54	-3.0277e+002	-7.0524e+003	7.0589e+003	-92.46	3.1198e+002
16	16	1222.980	1.1082e+003	-1.7259e+004	1.7295e+004	-86.33	1.1154e+003	-1.7253e+004	1.7289e+004	-86.30	5.3096e+002
13	16	1224.055	-4.6394e+003	-3.0840e+004	3.1187e+004	-98.55	-4.6441e+003	-3.0074e+004	3.0431e+004	-98.78	1.6271e+003
4	17	1328.036	-4.8959e+002	-1.5810e+004	1.5818e+004	-91.77	-4.7949e+002	-1.5803e+004	1.5810e+004	-91.74	8.5516e+002
17	16	1328.803	1.5461e+003	-1.4661e+004	1.4742e+004	-83.98	1.5462e+003	-1.4659e+004	1.4741e+004	-83.98	2.9335e+002
6	17	2019.269	8.1564e+002	-1.3301e+004	1.3326e+004	-86.49	7.3874e+002	-1.3345e+004	1.3365e+004	-86.83	6.6888e+002
23	15	2023.802	4.2669e+003	-6.3248e+004	6.3392e+004	-86.14	4.1880e+003	-6.3244e+004	6.3382e+004	-86.21	4.7741e+003
26	10	2321.862	1.6011e+003	-2.9188e+004	2.9232e+004	-86.86	8.1739e+002	-3.0454e+004	3.0465e+004	-88.46	5.2745e+003
28	8	2322.335	2.8552e+003	-3.6135e+004	3.6247e+004	-85.48	1.7616e+003	-3.7289e+004	3.7331e+004	-87.30	6.3617e+003
22	15	2337.895	-1.3348e+003	-5.0815e+004	5.0833e+004	-91.50	-1.3637e+003	-5.0848e+004	5.0867e+004	-91.54	2.6546e+003

TABLE 3. C-Plate Example: Modal Scaling Statistics

6. Typical Statistics: Bridge Example

For this example, impact data from a civil bridge structure test (15x55) has been used. The FRF data was processed using a Z-Domain Rational Fraction Polynomial Algorithm (RFP-Z) with all data between 5 and 30 Hz included. The model order range used was 2 to 20 with a generalized residual model including all terms from $(j\omega)^{-4}$ to $(j\omega)^{2}$. Further, low order alpha coefficient normalization has been used and full length residues have been calculated to be used in the CSSAMI procedure.

For the purposes of demonstration, all calculated poles (both consistent and computational) are included in the plots. As can be observed, though clearly present throughout the frequency range (Figures 6, 7 and 8) the non-realistic, computational poles, are not observed in the vicinity of the S-domain computed pole clusters (Figure 9)

The effect of prefiltering (by pole density alone) the information fed into the autonomous parameter estimation algorithm is shown in Figures 7 and 8. A comparison of no filtering (Figure 7) with prefiltering (Figure 8) reveals that more potential solutions are identified when the algorithm is allowed to match parameters on the basis of both pole and vector. A review of the results also indicates that the typical procedures for achieving clear consistency diagrams may actually be detrimental to the autonomous procedure because important information which didn't meet some arbitrary criteria was removed from consideration.

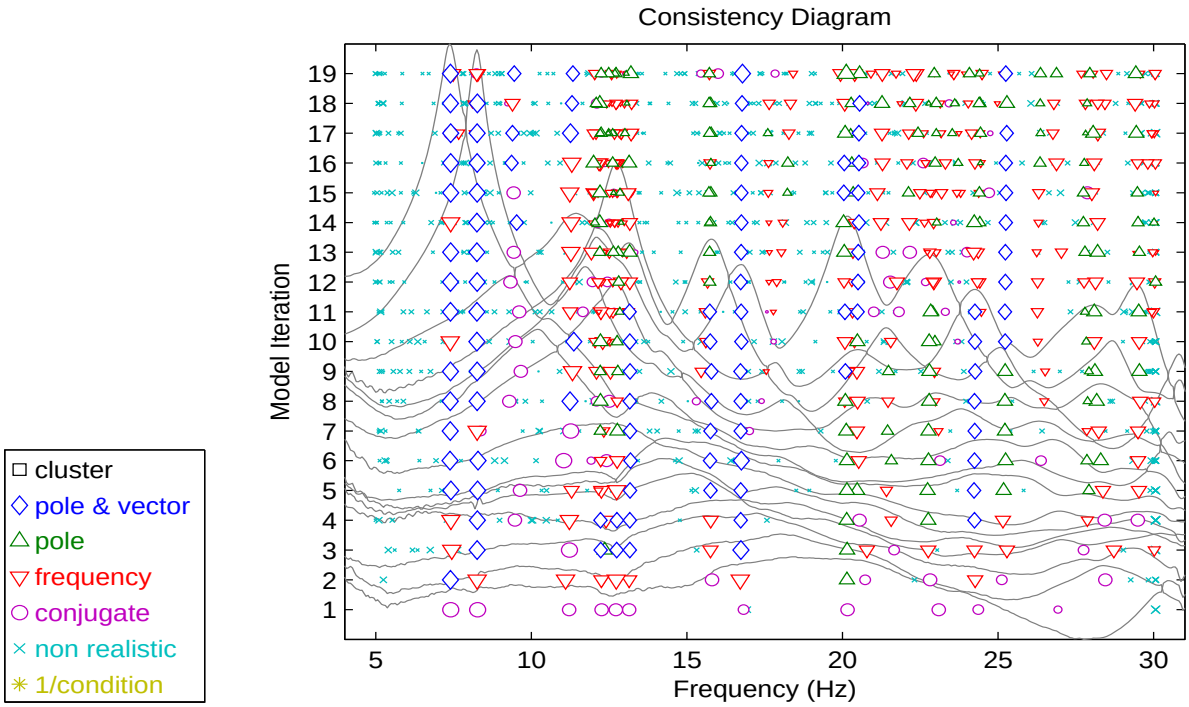

Figure 6. Bridge Example: Consistency Diagram - All Poles

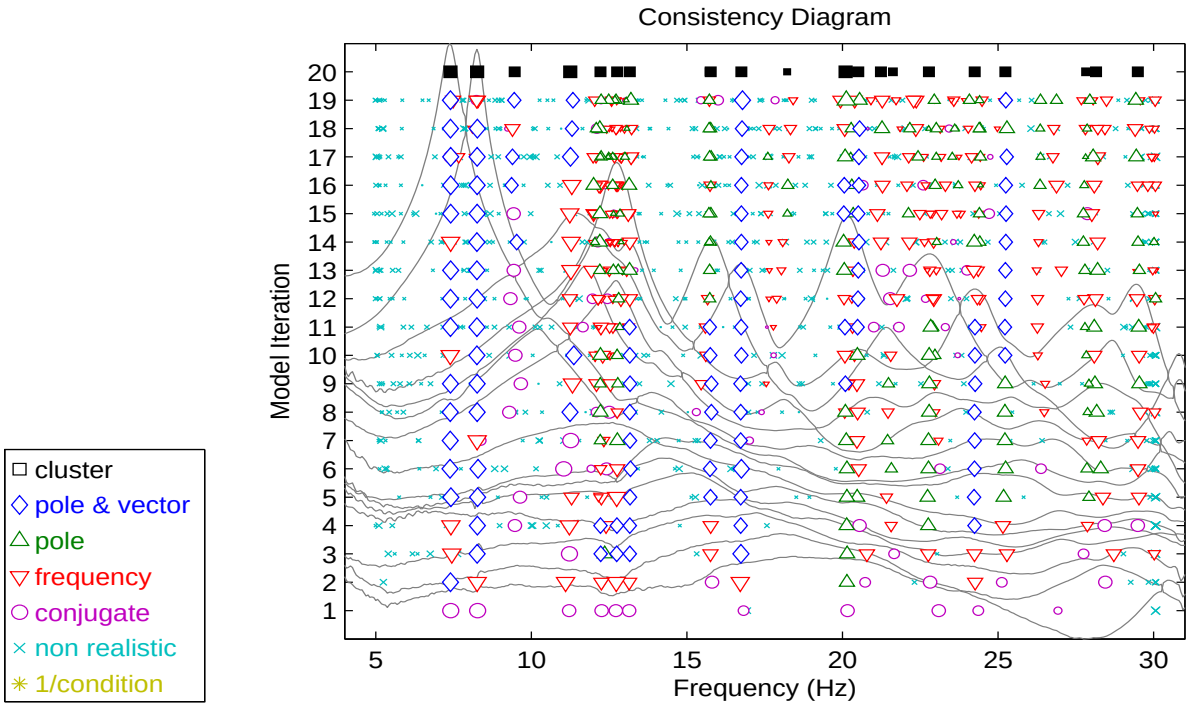

Figure 7. Bridge Example: Consistency Diagram - Pole Density = 1

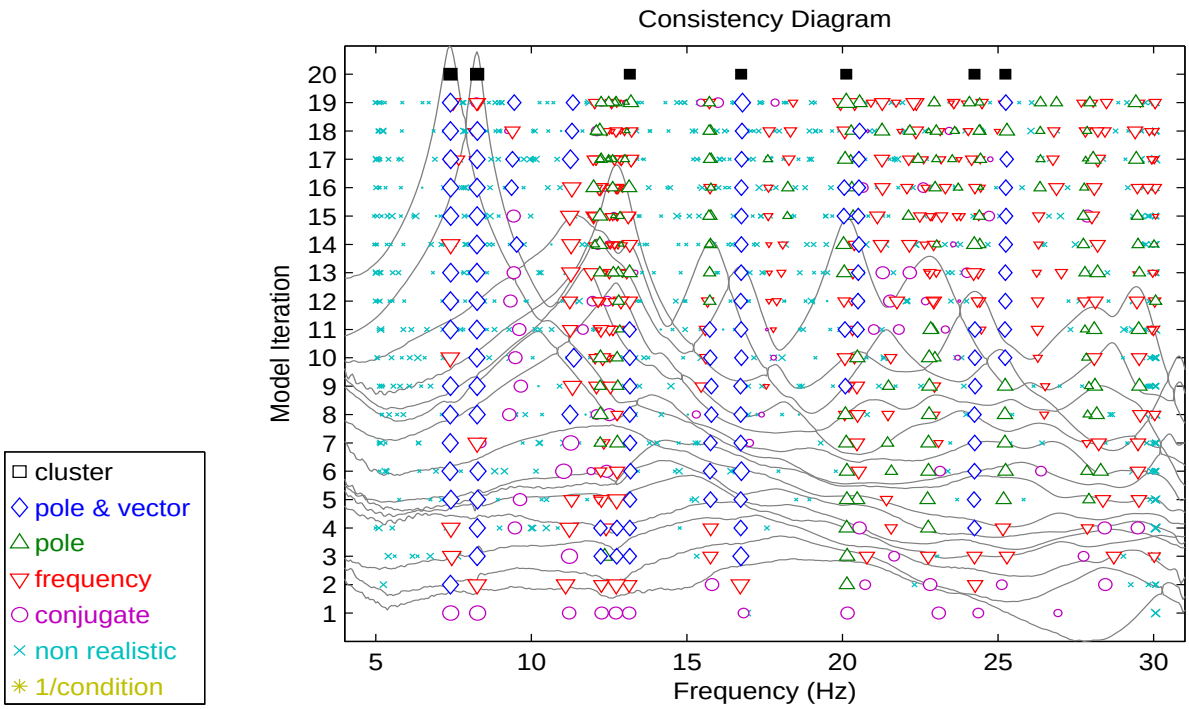

Figure 8. Bridge Example: Consistency Diagram - Pole Density = 4

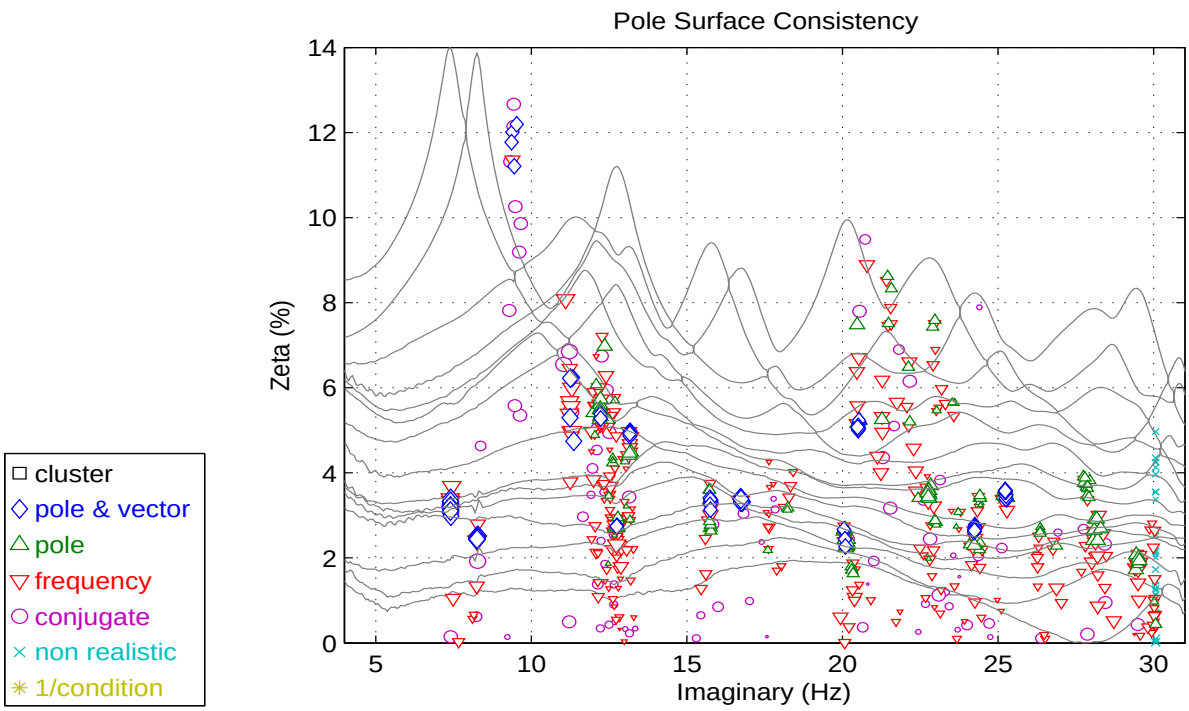

Figure 9. Bridge Example: Pole Surface Consistency - All Poles

The following plots, Figures 10 and 11, are a presentation of the statistical distribution of the pole information for four of the clusters shown in Figure 9. In Figures 10 and 11, the black stars are the original pole estimates and the red star represents the mean pole calculated from the cluster of pole estimates. The large red circle is the one standard deviation radius. The green

square is the indvidual standard deviations for the frequency and the damping. The blue circle is the pole as estimated from the pole weighted (state extended) vector.

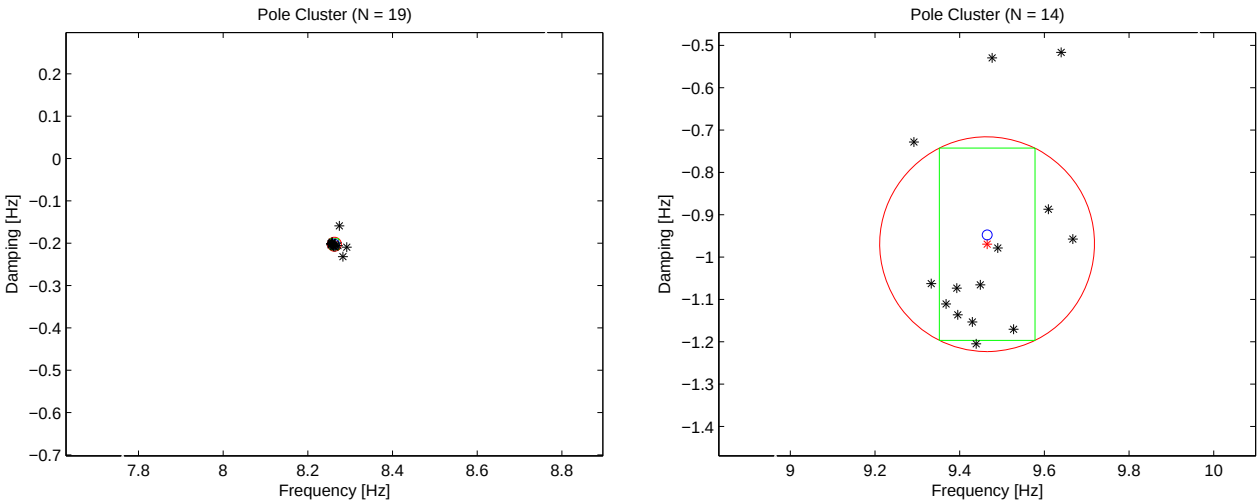

Figure 10. Bridge Example: 8.3 Hz and 9.5 Hz Pole Clusters - Statistics

As in the circular plate example, again note that the scaling in Figures 10 and 11 is consistent so that larger diameter circles will represent more variability.

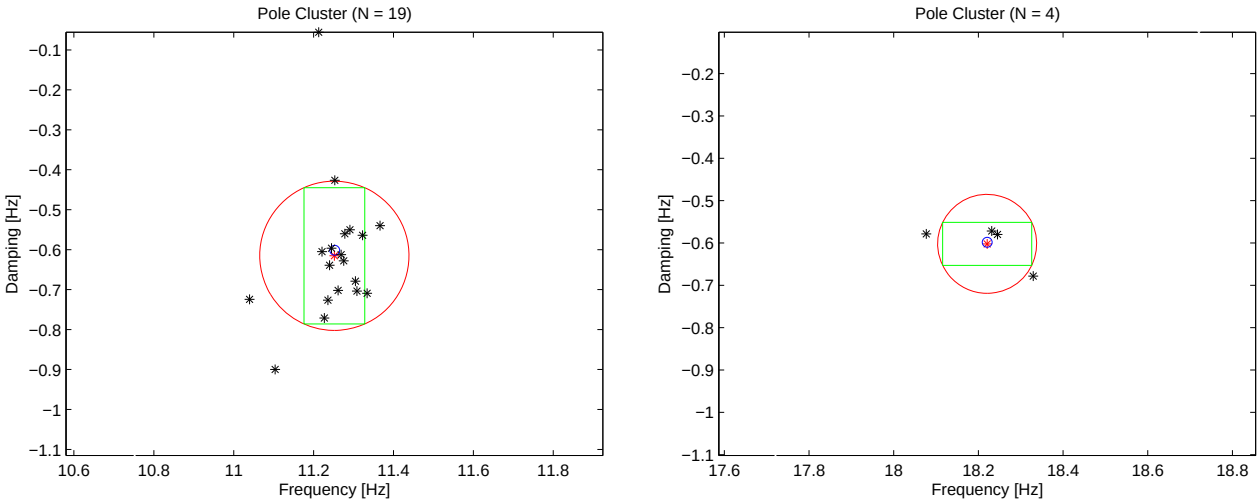

Figure 11. Bridge Example: 11.3 Hz and 18.2 Hz Pole Clusters - Statistics

The following (Tables 4, 5, and 6) provide a summary of the statistical information evaluated on the civil bridge structure.

Note that in Table 4 that there is little difference in the complex modal frequency estimated from the most significant pole weighted vector (columns 3 and 4) when compared to the mean values of the complex portions of the modal frequency (columns 5 and 6).

Index	Sample Size (N)	Frequency (Hz)	Damping (Hz)	Frequency (Hz) (Mean)	Damping (Hz) (Mean)	Std. Dev. (Hz)
3	19	7.406	-0.2404	7.406	-0.2405	0.0169
2	19	8.263	-0.2022	8.263	-0.2023	0.0165
21	14	9.465	-0.9480	9.465	-0.9698	0.2536
8	19	11.253	-0.6015	11.252	-0.6154	0.1868
35	6	12.224	-0.6669	12.224	-0.6670	0.0182
32	8	12.756	-0.3504	12.756	-0.3508	0.0295
11	17	13.162	-0.6187	13.162	-0.6191	0.0555
41	6	15.729	-0.5557	15.729	-0.5559	0.0294
30	10	15.765	-0.5059	15.765	-0.5065	0.0373
6	19	16.747	-0.5606	16.748	-0.5607	0.0282
43	4	18.221	-0.5984	18.221	-0.6022	0.1169
16	13	20.100	-0.4803	20.099	-0.4817	0.0501
37	4	20.323	-0.3610	20.323	-0.3616	0.0476
14	16	20.507	-1.1457	20.507	-1.1594	0.1925
39	5	21.236	-1.0438	21.236	-1.0540	0.1930
31	5	21.624	-1.5834	21.623	-1.5995	0.2917
25	10	22.778	-0.7731	22.778	-0.7754	0.0856
28	10	24.247	-0.6431	24.247	-0.6433	0.0238
9	17	25.235	-0.8774	25.235	-0.8777	0.0452
18	14	27.824	-1.0196	27.824	-1.0211	0.0749
23	8	28.150	-0.7324	28.151	-0.7365	0.1076
22	9	29.492	-0.4523	29.492	-0.4654	0.1662

TABLE 4. Bridge Example: Modal Frequency Statistics

In Table 5, again note that the fraction of normalized residual (columns 4-7) are all quite small indicated that only one mode has been found in each cluster and that the modal vectors in the cluster were essentially the same modal vector with minimal differences (noise).

Index	N	Frequency (Hz)	NMVR1	NMVR2	NSVR3	NSVR4
3	19	7.406	0.0008	0.0144	0.0009	0.0144
2	19	8.263	0.0022	0.0383	0.0022	0.0379
21	14	9.465	0.0056	0.0579	0.0079	0.0863
8	19	11.253	0.0053	0.0783	0.0064	0.0914
35	6	12.224	0.0357	0.1690	0.0357	0.1690
32	8	12.756	0.0193	0.1446	0.0194	0.1447
11	17	13.162	0.0087	0.0956	0.0089	0.0966
41	6	15.729	0.0238	0.1146	0.0238	0.1148
30	10	15.765	0.0075	0.0703	0.0076	0.0698
6	19	16.747	0.0039	0.0637	0.0039	0.0649
43	4	18.221	0.0536	0.1532	0.0540	0.1540
16	13	20.100	0.0143	0.1667	0.0143	0.1661
37	4	20.323	0.0408	0.1401	0.0410	0.1402
14	16	20.507	0.0104	0.1236	0.0104	0.1237
39	5	21.236	0.0394	0.1621	0.0406	0.1674
31	5	21.624	0.0385	0.1470	0.0411	0.1553
25	10	22.778	0.0116	0.0884	0.0120	0.0898
28	10	24.247	0.0067	0.0462	0.0068	0.0469
9	17	25.235	0.0072	0.0901	0.0073	0.0916
18	14	27.824	0.0124	0.1356	0.0125	0.1351
23	8	28.150	0.0180	0.1294	0.0183	0.1322
22	9	29.492	0.0238	0.1501	0.0242	0.1549

TABLE 5. Bridge Example: Modal Vector Statistics

In Table 6, again note that the method used to compute the modal scaling, Modal A, had little effect on the estimates of Modal A. Also of note in this case, is the phase angle of all Modal A terms is around ninety degrees. The sign on the phase angle and the closeness to ninety degrees (rather than zero degrees) is a function of the scaling method chosen for the modal vector.

Index	N	Frequency (Hz)	ModalA (State Vector)				ModalA (Mean from cluster)				
			real	imaginary	magnitude	phase	real	imaginary	magnitude	phase	Std. Dev.
3	19	7.406	-4.0159e+005	4.0857e+006	4.1054e+006	95.61	-4.0048e+005	4.0871e+006	4.1067e+006	95.60	1.3311e+005
2	19	8.263	-4.1063e+005	6.0755e+006	6.0893e+006	93.87	-3.7199e+005	6.0650e+006	6.0764e+006	93.51	7.2975e+005
21	14	9.465	5.1606e+006	1.3773e+007	1.4708e+007	69.46	6.6001e+006	1.4598e+007	1.6021e+007	65.67	4.9262e+006
8	19	11.253	-1.0672e+006	8.9505e+006	9.0139e+006	96.80	-2.6399e+006	1.2408e+007	1.2686e+007	102.01	1.5630e+007
35	6	12.224	-2.3579e+006	1.3038e+007	1.3250e+007	100.25	1.5347e+006	1.1077e+007	1.1183e+007	82.11	4.6407e+006
32	8	12.756	-3.7527e+005	8.6561e+006	8.6642e+006	92.48	-4.7815e+005	8.9063e+006	8.9191e+006	93.07	1.2880e+006
11	17	13.162	2.9493e+006	1.5389e+007	1.5669e+007	79.15	1.6985e+006	1.2154e+007	1.2272e+007	82.04	3.9192e+006
41	6	15.729	-1.1722e+006	9.6343e+006	9.7054e+006	96.94	-1.1850e+006	9.6038e+006	9.6766e+006	97.03	1.2406e+006
30	10	15.765	-1.2004e+006	1.0006e+007	1.0078e+007	96.84	-1.1961e+006	1.0022e+007	1.0093e+007	96.81	6.1042e+005
6	19	16.747	-1.5250e+006	1.4429e+007	1.4510e+007	96.03	-1.5462e+006	1.4422e+007	1.4504e+007	96.12	8.2761e+005
43	4	18.221	6.9955e+007	2.0003e+008	2.1191e+008	70.72	7.0403e+007	1.9977e+008	2.1181e+008	70.59	1.3292e+007
16	13	20.100	-3.4965e+005	1.9290e+007	1.9293e+007	91.04	-2.5684e+005	1.9387e+007	1.9388e+007	90.76	2.0555e+006
37	4	20.323	-1.6061e+007	7.0883e+007	7.2680e+007	102.77	-1.7005e+007	6.9200e+007	7.1258e+007	103.81	1.5557e+007
14	16	20.507	-3.8264e+005	2.2225e+007	2.2228e+007	90.99	4.2840e+006	2.3277e+007	2.3668e+007	79.57	9.5171e+006
39	5	21.236	-3.3460e+007	2.8599e+007	4.4017e+007	139.48	-3.6040e+007	2.5029e+007	4.3878e+007	145.22	1.2696e+007
31	5	21.624	3.4080e+006	1.6113e+007	1.6469e+007	78.06	2.4648e+006	1.2889e+007	1.3122e+007	79.17	3.3533e+006
25	10	22.778	-1.0823e+006	1.0448e+007	1.0504e+007	95.91	-1.0941e+006	1.0530e+007	1.0587e+007	95.93	1.1060e+006
28	10	24.247	-1.8809e+006	1.9384e+007	1.9476e+007	95.54	-1.8907e+006	1.9378e+007	1.9470e+007	95.57	5.5546e+005
9	17	25.235	-7.2001e+005	2.7462e+007	2.7471e+007	91.50	-8.3424e+005	2.6949e+007	2.6962e+007	91.77	2.2048e+006
18	14	27.824	-2.1672e+006	1.3965e+007	1.4132e+007	98.82	-2.0185e+006	1.4180e+007	1.4323e+007	98.10	3.0505e+006
23	8	28.150	-1.3309e+007	2.1013e+007	2.4873e+007	122.35	-1.1342e+007	2.1981e+007	2.4735e+007	117.29	7.1619e+006
22	9	29.492	-1.0596e+007	1.9653e+007	2.2327e+007	118.33	-1.2479e+007	2.3212e+007	2.6353e+007	118.26	1.5348e+007

TABLE 6. Bridge Example: Modal Scaling Statistics

7. Summary and Future Work

This paper has presented the statistics of the estimated modal parameters that are a direct result of a new development in autonomous modal parameter estimation. The proposed method represents an important philosophical and paradigm shift in the process of identifying potentially valid modal parameters. Instead of trying to get only (statistically and numerically) well estimated modes and then adding in the marginally estimated modes from the original estimates from the consistency diagram, the autonomous MPE methodology attempts to estimate all possible modes and use the statistics of the solutions to eliminate the marginal estimates. It has also been observed that, contrary to conventional wisdom, the techniques for producing clear consistency (stabilization) diagrams are at odds with the autonomous procedure. This is in part due to the nature of the consistency diagram where frequency and damping consistency tolerances used in the consistency diagram causes somewhat inconsistent estimates to be eliminated from consideration before the vector consistency can be evaluated. The temporal-spatial nature of the CSSAMI procedure allows these helpful results to be retained by the solution procedure.

One obvious extension of this work is to initiate the autonomous modal parameter estimation process using consistency diagram information from several different methods rather than just one. The inclusion of more estimates from differing algorithms may mean better consistency of estimates for some modes that are poorly estimated from a given algorithm.

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