Autonomous Modal Parameter Estimation: Methodology

A.W.Phillips, R.J. Allemang, D.L. Brown
Structural Dynamics Research Laboratory
School of Dynamic Systems
College of Engineering and Applied Science
University of Cincinnati
Cincinnati, OH 45221-0072 USA
Email: Allyn.Phillips@UC.EDU

ABSTRACT

Traditionally, the estimation of modal parameters from a set of measured data has required significant experience. However, as the technology has matured, increasingly, analysis is being performed by less experienced engineers or technicians. To address this development, frequently software solutions are focusing upon either wizard-based or autonomous/semi-autonomous approaches. A number of autonomic approaches to estimating modal parameters from experimental data have been proposed in the past. In this paper, this history is revewed and a technique suitable for either approach is presented. By combining traditional modal parameter estimation algorithms with a-priori decision information, the process of identifying the modal parameters (frequency, damping, mode shape, and modal scaling) can be relatively simple and automated. Examples of the efficacy of this technique are shown for both laboratory and real-world applications in a related paper.

Nomenclature

N_i = Number of inputs.
N_o = Number of outputs.
N_S = Short dimension size.
N_L = Long dimension size
N = Number of modal frequencies.
λ_r = S domain polynomial root.
λ_r = Complex modal frequency (rad/sec).
$\lambda_r = \sigma_r + j\,\omega_r$
σ_r = Modal damping.
ω_r = Damped natural frequency.
z_r = Z domain polynomial root.
$\{\psi_r\}$ = Base vector (modal vector).
$\{\phi_r\}$ = Pole weighted base vector (state vector).
$[A_r]$ = Residue matrix, mode r.
[I] = Identity matrix.
t_i = Discrete time (sec).
ω_i = Discrete frequency (rad/sec).
s_i = Generalized frequency variable.
$x(t_i)$ = Response function vector $(N_o \times 1)$).
$X(\omega_i)$ = Response function vector $(N_o \times 1)$).
$f(t_i)$ = Input function vector $(N_i \times 1)$)
$F(\omega_i)$ = Input function vector $(N_i \times 1)$).
$[h(t_i)]$ = IRF matrix $(N_o \times N_i)$).
$[H(\omega_i)]$ = FRF matrix $(N_o \times N_i)$).
$[\alpha]$ = Denominator polynomial matrix coefficient.
$[\beta]$ = Numerator polynomial matrix coefficient.
m = Model order for denominator polynomial.
n = Model order for numerator polynomial.
v = Model order for base vector.
r = Mode number.

1. Introduction

The desire to estimate modal parameters automatically, once a set or multiple sets of test data are acquired, has been a subject of great interest for more than 40 years. In the 1960s, when modal testing was limited to analog test methods, several researchers were exploring the idea of an automated test procedure for determining modal parameters [1-3]. Today, with the increased memory and compute power of current computers used to process test data, an automated or autonomous, modal parameter estimation procedure is entirely possible and is being attempted by numerous researchers.

Before proceeding with a discussion of autonomous modal parameter estimation, some philosophy and definitions regarding

what is considered autonomous is required. In general, autonomous modal parameter estimation refers to an automated procedure that is applied to a modal parameter estimation algorithm so that no user interaction is required once the process is initiated. This typically involves setting a number of parameters or thresholds that are used to guide the process in order to exclude solutions that are not acceptable to the user. When the procedure finishes, a set of modal parameters is identified that can then be reduced or expanded if necessary. The goal is that no further reduction, expansion or interaction with the process will be required.

The larger question concerning autonomous modal parameter estimation is the intended user. Is the autonomous modal parameter estimation procedure expected to give results sufficiently robust for the novice user? This implies that the user could have no experience with modal analysis and, therefore, have no experiential judgement to use in assessing the quality of the results. The use of the term *wizard* implies that this is the desired situation. In contrast, the user could be very knowledgeable in the theory and experienced. For this case, the autonomous modal parameter estimation procedure is simply an efficient mechanism for sorting a very large number of solutions into a final set of solutions that satisfies a set of criteria and thresholds that are acceptable to the user. This user is the assumed reader for the purposes of this discussion.

In order to discuss autonomous modal parameter estimation, some background is needed to clarify terminology and methodology. First, a brief history and overview of documented autonomous modal parameter estimation methods is presented. Second, an overview of some of the tools or semi-autonomous methods that are useful as part of the autonomous modal parameter estimation procedure are reviewed. Finally, the general procedure for an autonomous modal parameter estimation methods that is based upon consistent state vectors, referred to as **Common Statstical Subspace Autonomous Mode Identification (CSSAMI),** is presented followed by a number of examples. Note that much of the background of the following discussion will be based upon the Unified Matrix Polynomial Algorithm (UMPA) developed by the authors and described in a number of other papers [4-8] .

2. Background: Autonomous Modal Parameter Estimation

The interest in automatic modal parameter estimation methods has been documented in the literature since at least the mid 1960s when the primary modal method was the analog, force appropriation method [1-3]. Following that early work, there has been a continuing interest in autonomous methods [9-28] that, in most cases, have been procedures that are formulated based upon a specific modal parameter estimation algorithm like the Eigensystem Realization Algorithm (ERA), the Polyreference Time Domain (PTD) algorithm or more recently the Polyreference Least Squares Complex Frequency (PLSCF) algorithm or the commercial version of the PLSCF, the PolyMAX ® method.

Each of these past procedures have shown some promise but have not yet been widely adopted. In many cases, the procedure focussed on a single modal parameter estimation algorithm and did not develop a general procedure. Most of the past procedural methods focussed on pole density but depended on limited modal vector data to identify correlated solutions. Currently, due to increased computational speed and larger availability of memory, procedural methods can be developed that were beyond the computational scope of available hardware only a few years ago. These methods do not require any initial thresholding of the solution sets and rely upon correlation of the vector space of thousands of potential solutions as the primary identification tool. With the addition to any modal parameter estimation algorithm of the concept of pole weighted vector, the length, and therefore the sensitivity, of the these extended vectors provides an additional tool that appears to be very useful in autonomous modal parameter estimation.

3. Numerical Tools - Autonomous Modal Parameter Estimation

The development of many of the past procedures and any new proposed procedure depends heavily on a number of numerical tools that have been developed over the last twenty years or so [29-55]. These tools are currently used by many algorithms as a user interaction tool or semi-autonomous tool to assist the user in picking an appropriate set of modal parameters. These tools are described in great detail in the literature and are summarized briefly in the next several sections.

3.1 Consistency Diagrams

For the last thirty years, modal parameter estimation based upon experimental data, primarily frequency response functions (FRFs), has utilized some form of error chart and/or stabilization diagram to visualize and assist in the determination of the correct modal frequencies [46-49,53-54]. The conceptual basis of the stabilization diagram is that distinct and unique modal frequencies can be identified by comparing the roots of a characteristic polynomial when the model order of the characteristic polynomial is increased or the subspace is altered in a systematic manner. The stabilization or consistency diagram is based upon successive solutions of Equation (1) or Equation (2) for different values of the maximum model order m. If the roots are consistent as the model order is increased, these roots are identified as modal frequencies. If the roots are inconsistent,

these roots are associated with noise on the data and are discarded. Since there is generally much more FRF data available than is needed to solve for the number of modal parameters of interest, the characteristic polynomial is normally estimated in a least squares sense and can be reformulated from the measured data for each model order [4-5].

Frequency Domain Algorithms:

$$\left| [\alpha_m]\, s^m + [\alpha_{m-1}]\, s^{m-1} + [\alpha_{m-2}]\, s^{m-2} + \ldots\ldots\ldots + [\alpha_0] \right| = 0 \tag{1}$$

where:

$$s_r = \lambda_r \qquad \lambda_r = \sigma_r + j\,\omega_r$$

Time Domain Algorithms:

$$\left| [\alpha_m]\, z^m + [\alpha_{m-1}]\, z^{m-1} + [\alpha_{m-2}]\, z^{m-2} + \ldots\ldots\ldots + [\alpha_0] \right| = 0 \tag{2}$$

where:

$$z_r = e^{\lambda_r \Delta t} \qquad \lambda_r = \sigma_r + j\,\omega_r$$

$$\sigma_r = \mathrm{Re}\left[\frac{\ln z_r}{\Delta t}\right] \qquad \omega_r = \mathrm{Im}\left[\frac{\ln z_r}{\Delta t}\right]$$

In most implementations, the **stabilization diagram** or **consistency diagram** is presented as a two dimensional plot with frequency on the abscissa and characteristic polynomial model order on the ordinate. A plot of the summed magnitude of all of the FRFs, or a plot of one of the mode indicator functions (complex mode indicator function (CMIF) or multivariate mode indicator function (MvMIF)), is placed in the background for reference. For each model order, symbols are plotted along the frequency axis wherever a root of the characteristic polynomial has been estimated. Historically, the characteristic polynomial had scalar coefficients (single reference data) so the roots are complex valued and have both frequency and damping information. The original symbols were used to indicate, in increasing importance, Level 1: a pole was found, Level 2: a pole was found with an associated complex conjugate, Level 3: Level 2 plus the damping was realistic (negative real part of pole), Level 4: Level 3 plus a damped natural frequency (imaginary part of pole) consistency within a specified percentage (normally 1 percent) and Level 5: Level 4 plus a damping consistency with a specified percentage (normally 5 percent).

As multiple reference parameter estimation was developed, this led to matrix coefficient, characteristic polynomial equations which created further evaluation criteria, Level 6: Level 5 plus a vector consistency within a specified modal assurance criterion (MAC) value (normally 0.95). Since the vector that is associated with the matrix coefficient, characteristic polynomial may be different sizes (function of N_S or N_L), a variation of Level 6 involves an additional solution for the vector at the largest dimension N_L so that a more statistically significant comparison of vectors can be utilized. Finally, numerical conditioning can be evaluated for each solution and if the numerical conditioning is approaching a limit based upon the accuracy of the data or the numerical limitations of the computer algorithm or word size, Level 7 can be added to indicate a possible numerical problem. In general, all of the specified values that indicate consistency at each level can be user defined and are referred to as the stability or consistency tolerances.

This information generates a consistency diagram, for a simple circular plate structure, that looks like Figure 1. With all of the symbols presented, the consistency diagram for even this simple structure can be very complicated. Figures 1 and 2 demonstrate two different presentations of consistency diagrams based upon different presentations of the characteristic matrix coefficient polynomial information. In both figures, an average autopower of the measured data is plotted on the consistency diagram in the background for reference.

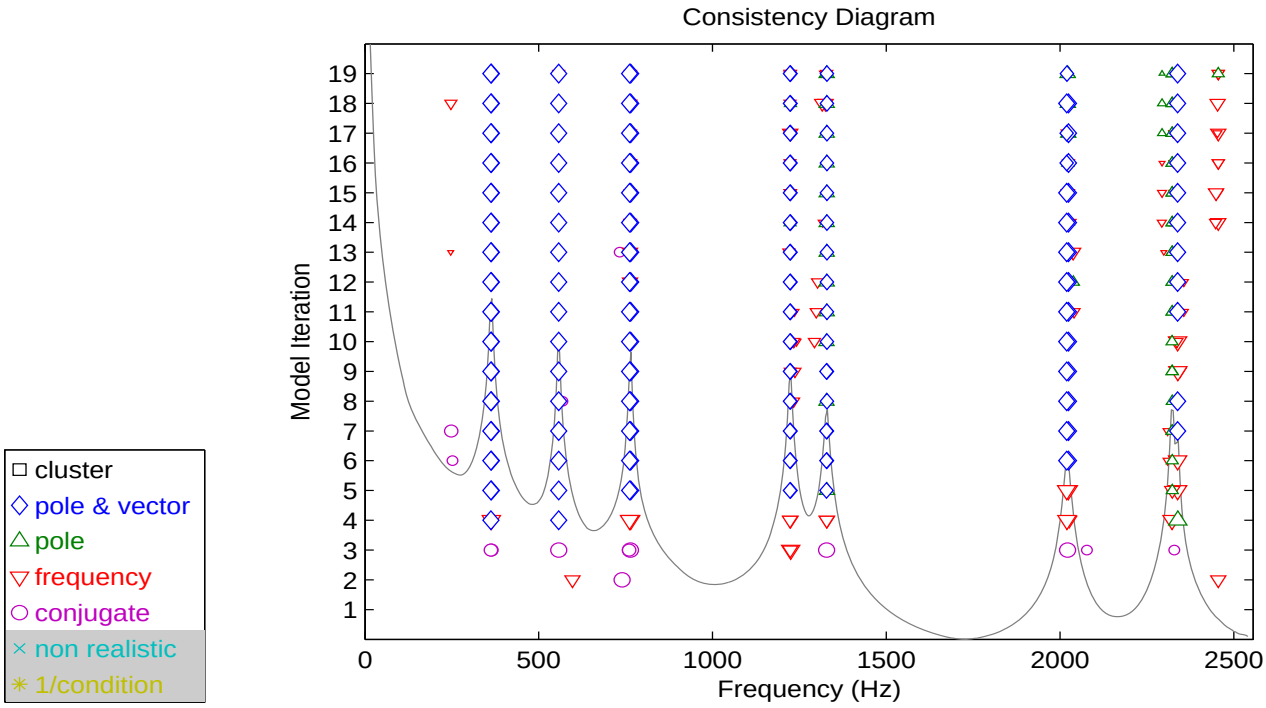

Figure 1. Typical Consistency Diagram

Figure 2 presents the consistency diagram using an entirely different method that involves using the modal assurance criterion (MAC) to compare vectors from successive solutions of the characteristic polynomial equation. Rather than comparing the unscaled modal vectors (base vectors) directly, a pole weighted vector of v-th order is constructed for each solution and compared to similarly constructed pole weighted vectors for the previous solution. Further explanation of the pole weighted vector is provided in Section 3.3 and can also be found in the literature [46-47].

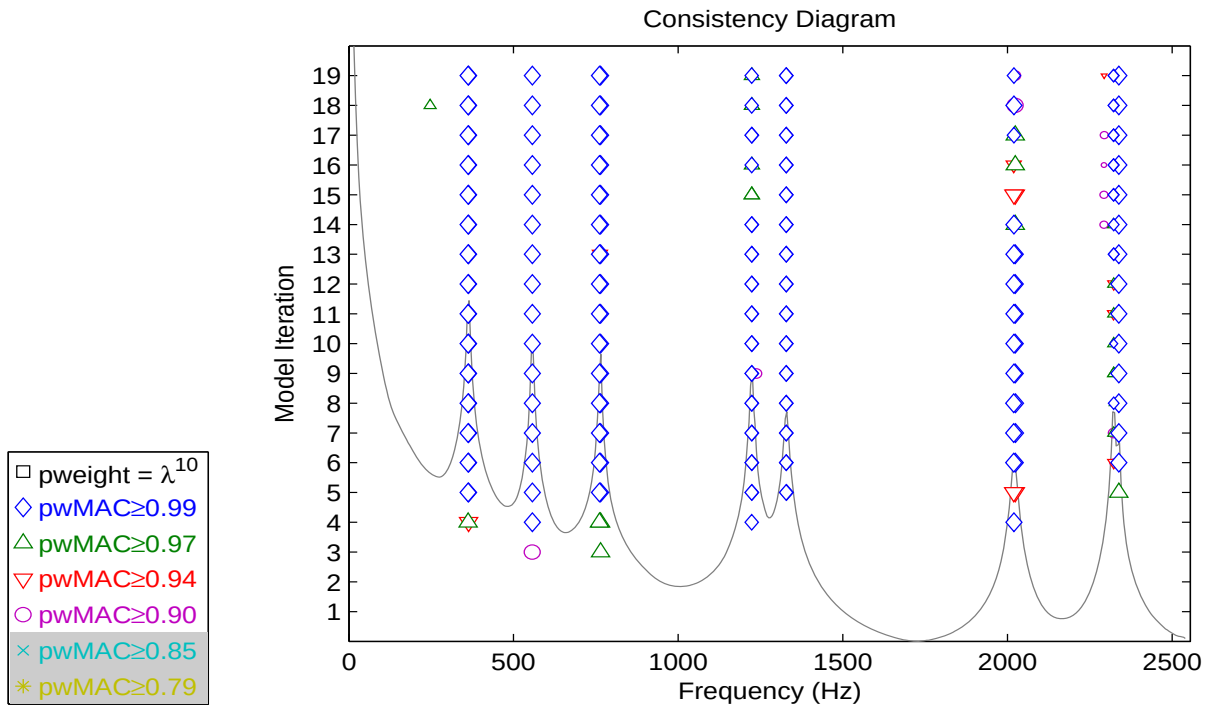

Figure 2. Alternate Consistency Diagram - Pole Weighted MAC

3.1.1 Clear Consistency Diagrams

A number of different methods can be used to generate the consistency diagram that will impact the clarity of the consistency diagram. A number of recent papers [46-47,53-54] have identified the effects of changing the consistency tolerances on the resultant consistency diagrams, yielding a clearer presentation of symbols that indicate the presence of a structural mode of vibration. These methods can be combined with the coefficient normalization and consistency tolerances to generate a very clear diagram in most cases where the measured data has a reasonable match to linear, reciprocal system assumptions, observability issues and reasonable data noise levels. These methods have been used on a wide range of data cases in the automotive and aerospace application areas with good success. The methods include:

- Symbol Sizing Based Upon Normal Mode Criteria
- Complete or Incomplete Vector Comparisons
- Using Both Coefficient Normalization Methods
- Numerator and Denominator Model Order Variation
- Fixed Denominator and Numerator Order Variation
- Frequency Normalization Variation

3.2 Long Dimension Vector Solution

While many high order, matrix coefficient modal parameter estimation methods estimate unscaled modal vectors as part of the estimation of the poles, there is no reason to limit the length of the unscaled modal vector to the short dimension. Each short dimension vector can be used to estimate the unscaled vector for the long dimension as a part of the solution. This requires an extra solution step but, for each model order, requires very little additional computational effort. In this case, regardless of the method employed to estimate the modal frequencies, all unscaled modal vectors will be of length equal to the long dimension. No attempt to restrict the set of modal frequencies is used at this point in the procedure; all possible poles are included in this calculation. This set of unscaled vectors will include structural modal vectors and computational vectors. Sorting these vectors is left to a correlation procedure such as the modal assurance criterion (MAC) with a threshold (minimum MAC value). Note that this extra step can include the estimation of modal scaling so that scaled modal vectors and modal scaling values can be used as part of the evaluation process.

3.3 Pole Weighted Modal Vectors

When comparing base vectors, at either the short or the long dimension, a pole weighted base vector can be constructed independent of the original UMPA(m,n,v) procedure used to estimate the poles and base vectors. For a given order v of the pole weighted vector, the base vector and the associated pole can be used to formulate the pole weighted vector as follows:

$$\{\phi\}_r = \begin{Bmatrix} \lambda_r^v\{\psi\}_r \\ \cdot \\ \cdot \\ \cdot \\ \lambda_r^2\{\psi\}_r \\ \\ \lambda_r^1\{\psi\}_r \\ \\ \lambda_r^0\{\psi\}_r \end{Bmatrix}_r \tag{3}$$

The above formulation will be dominated by the high order terms if actual frequency units are utilized. Generalized frequency concepts (frequency normalization or z domain transform) are normally used to minimize this problem as follows:

$$\{\phi\}_r = \begin{Bmatrix} z_r^v\{\psi\}_r \\ \cdot \\ \cdot \\ \cdot \\ z_r^2\{\psi\}_r \\ \\ z_r^1\{\psi\}_r \\ \\ z_r^0\{\psi\}_r \end{Bmatrix}_r \tag{4}$$

$$z_r = e^{j*\pi*(\omega_r/\omega_{max})} = e^{j*\omega_r*\Delta t} \tag{5}$$

$$z_r^m = e^{j*\pi*m*(\omega_r/\omega_{max})} \tag{6}$$

In the above equations, Δt and ω_{max} can be chosen as needed to cause the positive and negative roots to wrap around the unit circle in the z domain without overlapping (aliasing). Normally, ω_{max} is taken to be five percent larger than the largest frequency identified in the roots of the matrix coefficient polynomial.

3.4 Pole Surface Consistency and Density

A number of other pole presentation diagrams, related to the consistency diagram, such as pole surface consistency and pole surface density diagrams have proven useful for identifying modal parameters [43-44,56-57] and may be more powerful than the consistency diagram alone. Generally, pole surface density diagrams are more powerful than consistency or stability diagrams at locating similar pole vector estimates from all of the possible solutions represented in the consistency diagram. All of the poles from all solutions involved in the consistency diagram are ploted in the second quadrant of the s plane. Pole estimates that are located within a two dimensional theshold from each other are defined as participating in a pole cluster or a dense region of estimated poles in the s plane. The poles that are compared on these pole surface diagrams are generally limited to the poles identified on the consistency diagram (if some symbols are omitted from the consistency diagram, these poles will not be included on the pole surface diagram. The distribution of the poles that participate in a pole cluster can be used to find a single pole-vector estimate and the distribution can be used to estimate statistics related to the variance in the pole estimate. An example of a pole surface consistency diagram is given in Figure 3 and the companion pole surface density diagram in Figure 4.

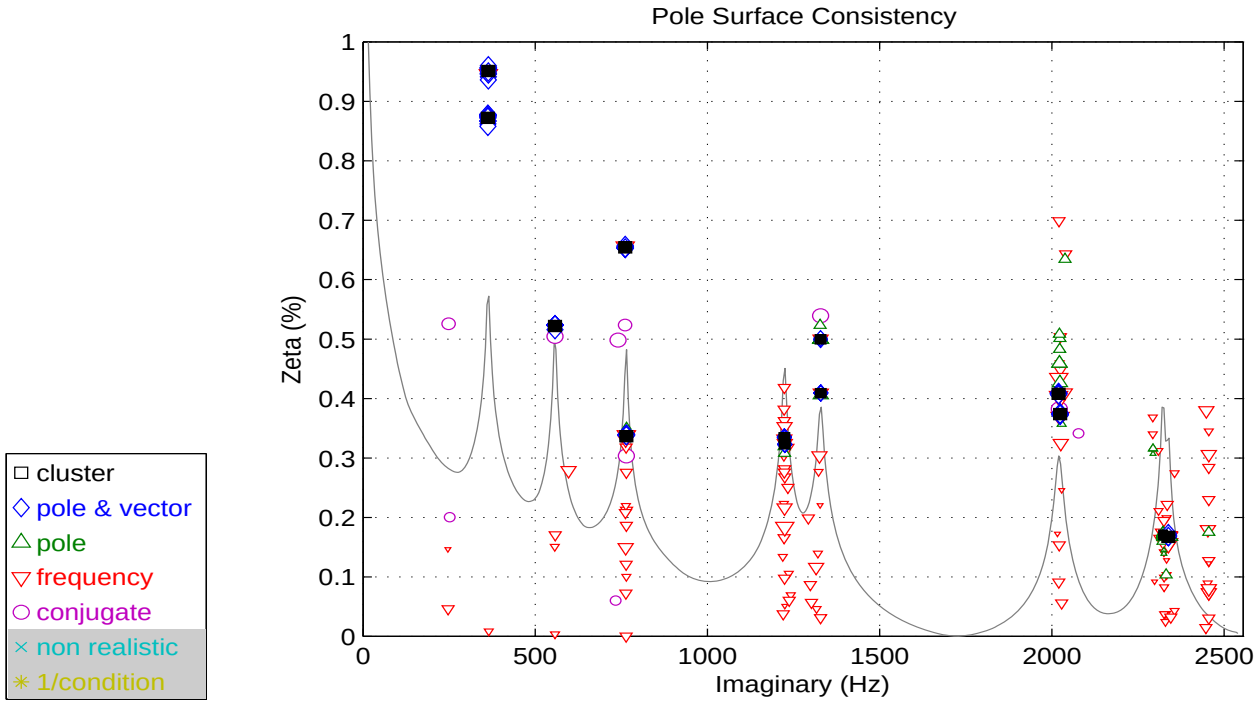

Figure 3. Pole Surface Consistency Diagram with Final Autonomous Estimates

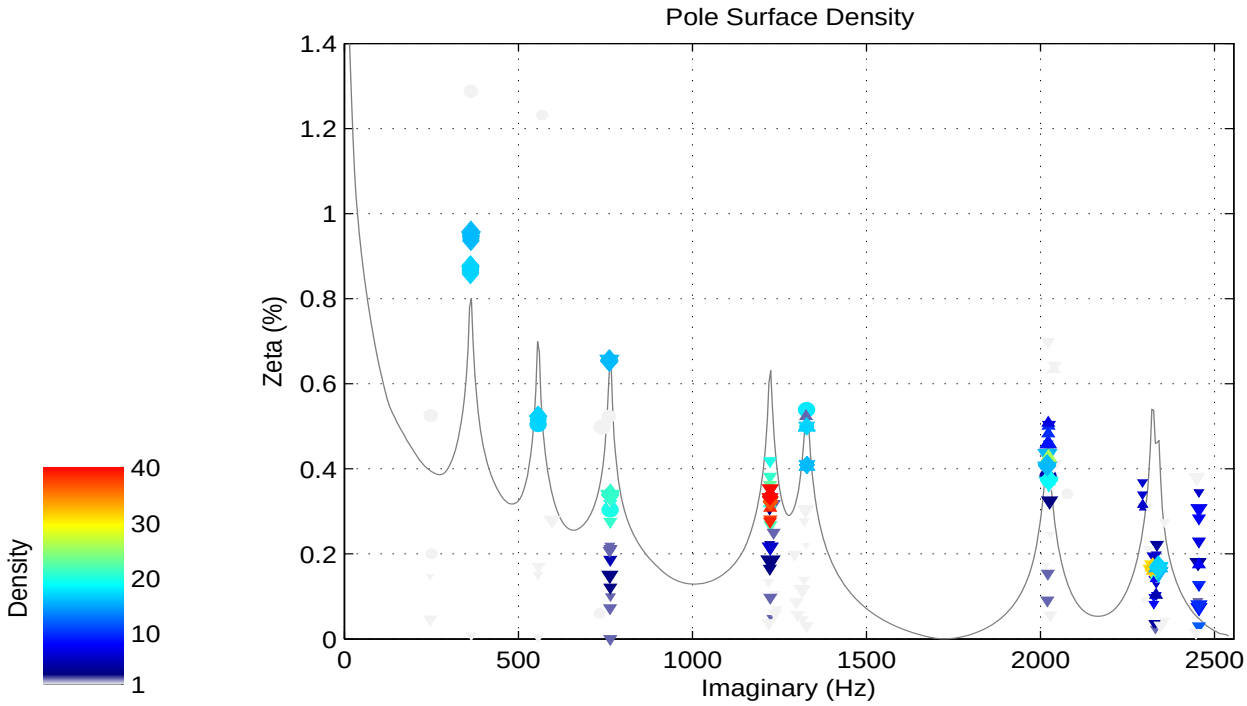

Figure 4. Pole Surface Density Diagram

Figures 3 and 4 presents the same information previously shown in Figures 1 and 2. Note that in all cases, suppressing the obvious spurious computational poles results in a substantially cleaner consistency diagram with clear indications of the pole locations. This translates to the Pole Surface Diagram as well. Note that suppressing spurious modes entails a risk that some

valid modes may be eliminated which may not be attractive behavior once autonomous modal parameter estimation procedures are utilized.

4. Autonomous Modal Parameter Estimation Method

The autonomous modal parameter estimation method developed and presented in the following is a general method that can be used with any algorithm that fits within the UMPA structure. A complete description of the Unified Matrix Polynomial Algorithm (UMPA) thought process can be found in a number of references developed by the authors [4-8, 57]. This means that this method can be applied to both low and high order methods with low or high order base vectors. This also means that most commercial algorithms could take advantage of this procedure. Note that high order matrix coefficient polynomials normally have coefficient matrices of dimension that is based upon the short dimension of the data matrix ($N_S \times N_S$). In these cases, it may be useful to solve for the long dimension modal vectors or to use pole weighted modal vectors. This will extend the temporal-spatial information in the N_S length base vector so that the vector will be more sensitive to change. This characteristic is what gives this autonomous method (CSSAMI) the ability to distinguish between computational and structural modal parameters.

The implementation of the autonomous modal parameter estimation for this method is detailed in the following sections.

4.1 Step 1: Develop a Consistency Diagram

Develop a consistency diagram using any UMPA solution method. Since this autonomous method utilizes a pole surface density plot, having a large number of iterations in the consistency diagram (due to model order, subspace iteration, starting times, equation normalization, etc.) will be potentially advantageous. The possibility of combining solutions from different consistency diagrams originating from different UMPA models is also a natural extension of this autonomous approach.

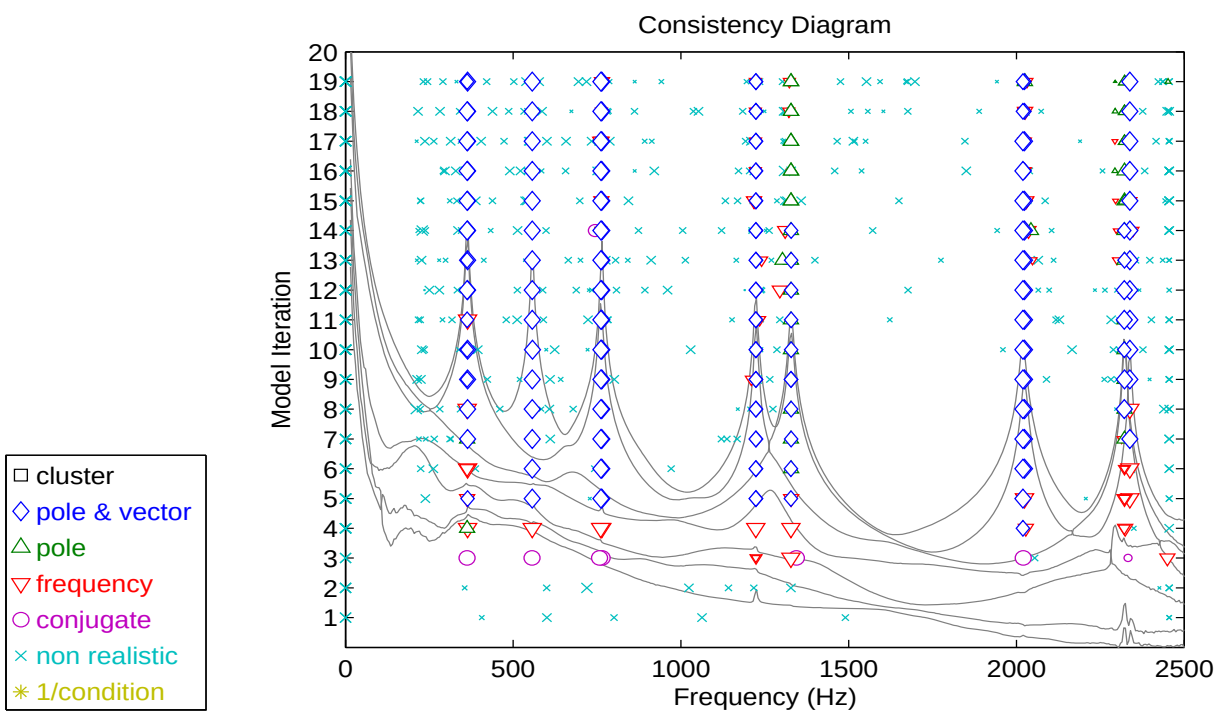

Figure 5. Consistency Diagram Showing All Poles

The larger the number of solutions (represented by symbols) in the consistency diagram, the more computation time and memory will be required. However, restricting the number of solutions using clear stabilization (consistency) methods may be counterproductive [59].

4.2 Step 2: Find Scaled Modal Vectors

If the UMPA method is high order (coefficient matrices of size $N_S \times N_S$), solve for the complete vector (function of N_L for all roots, structural and computational).

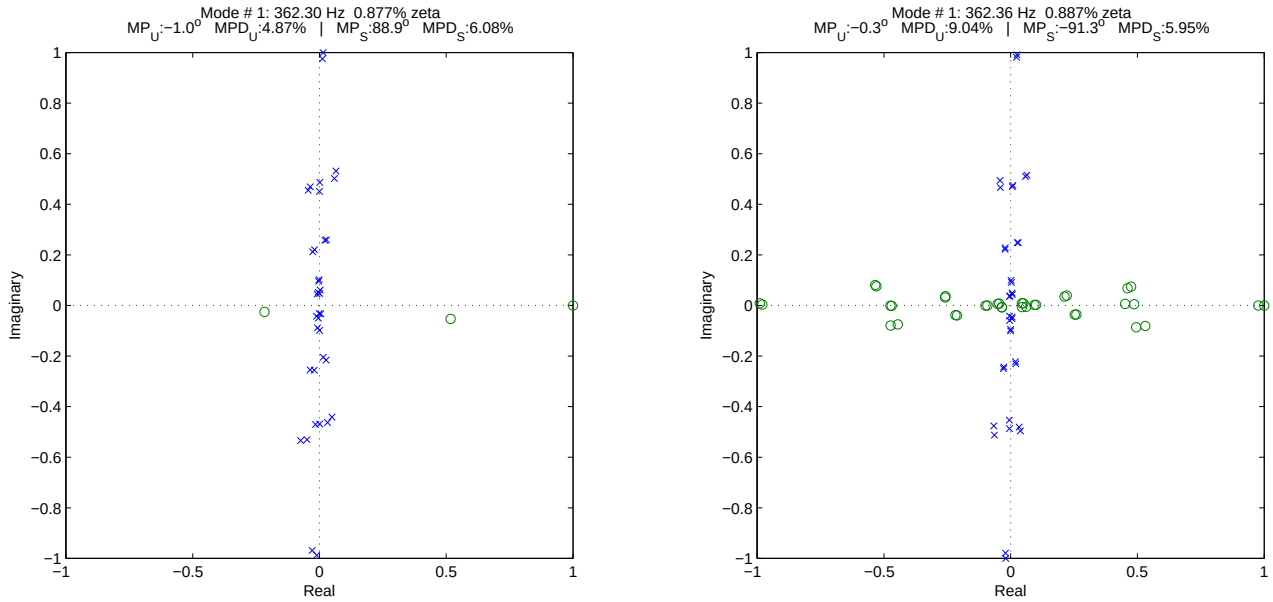

Figure 6. Typical Complex Plot of Unscaled and Scaled Modal Vectors

Figure 6 shows plots of a complex valued modal vector originating from PTD (left) and ERA (right) methods. The green circle symbols represent the normalized modal vector associated with the consistency diagram for one of the poles of one of the high order solutions. The blue x symbols represent the scaled (in terms of residue) full length vectors that can be computed, mode by mode, before continuing. The purpose in calculating the complete, scaled vector is two-fold. The first is the additional discrimination of the algorithm that results from the longer vector length (PTD example on the left). The second is the availability of the modal scaling information, used to calculate Modal A. This scaling information can be useful at the end of the solution process in order to asses the quality of the results (magnitude and phase of Modal A, for example). Even long basis vectors should be scaled so that this assessment information is available for all cases (ERA example on the right).

4.3 Step 3: Determine Pole Surface Density Clusters

Based upon the pole surface density threshold, identify all possible pole densities above some minimum value. This will be a function of the number of possible solutions represented by the consistency diagram. The pole density is defined for each pole as the number other poles within a specified complex tolerance radius. This range can be defined in terms of either absolute or relative frequency. Just as in Step 1, restricting the number of poles reduces both the time and the memory required for solution, but it also reduces the amount of information available to the algorithm. As a result, an overly restrictive threshold may be counterproductive. Figures 7 and 8 show the pole consistency plots (showing pole locations in the second quadrant of the s-plane) that are evaluated in this step.

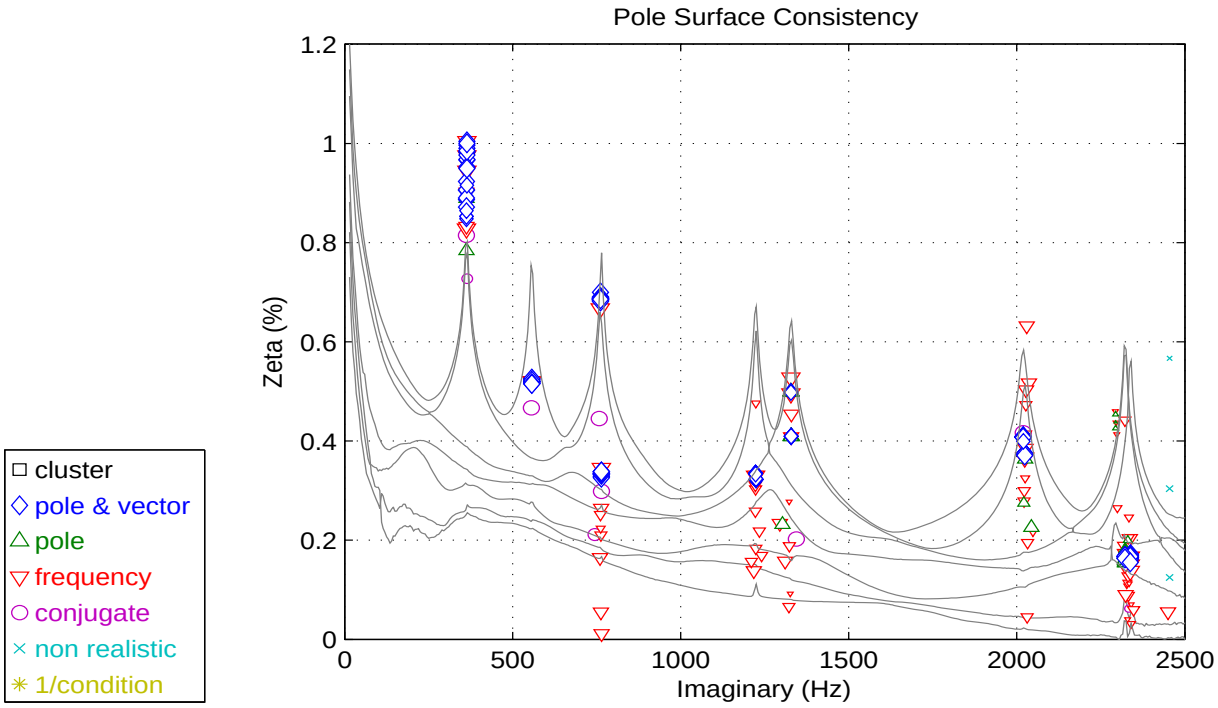

Figure 7. Pole Surface Consistency Showing All Poless

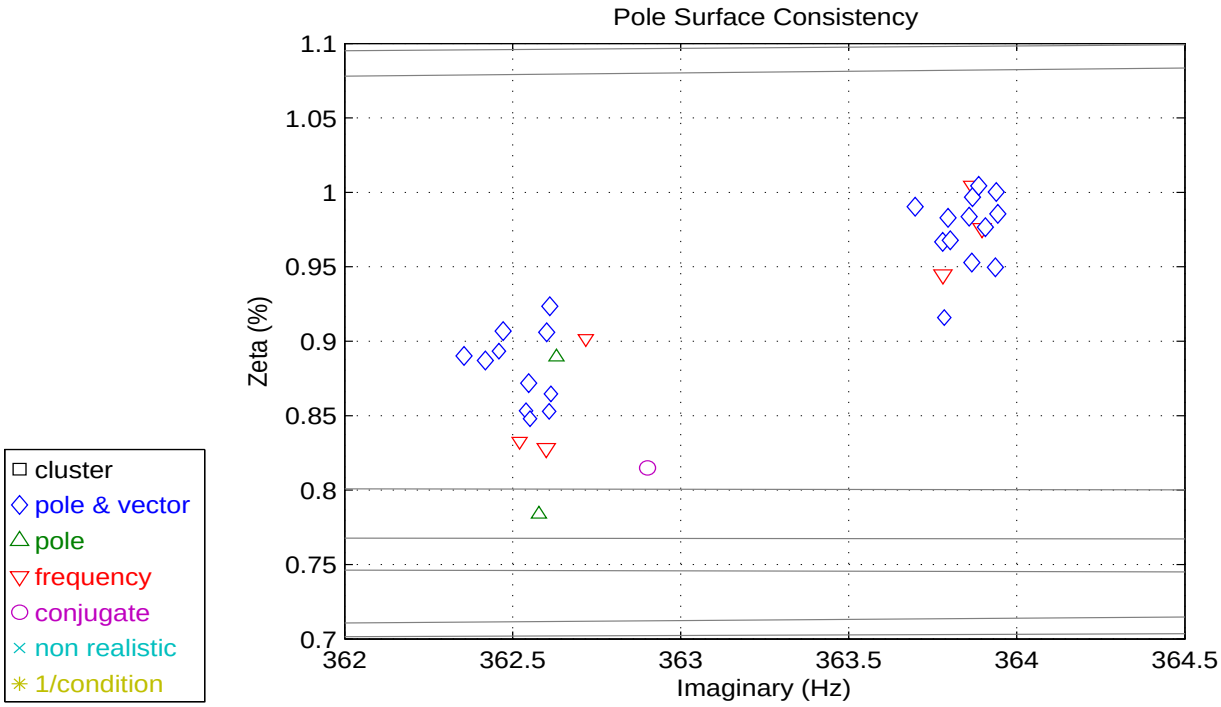

Figure 8. Zoomed Pole Clusters

4.4 Step 4: Construct Normalized Pole Weighted Vectors

Normalize each vector to unity length with dominant real part and then construct a predefined, higher order (typically 10th order), pole weighted vector for each solution [47].

$$\{\phi\}_r = \begin{Bmatrix} z_r^v\{\psi\}_r \\ \cdot \\ \cdot \\ \cdot \\ z_r^2\{\psi\}_r \\ z_r^1\{\psi\}_r \\ z_r^0\{\psi\}_r \end{Bmatrix}_r$$

4.5 Step 5: Calculate Auto-MAC of Normalized, Pole Weighted Vectors

Sort the normalized pole weighted vectors created in Step 4 into frequency order based upon damped natural frequency (ω_r). Then calculate the Auto-MAC matrix for all pole weighted vectors.

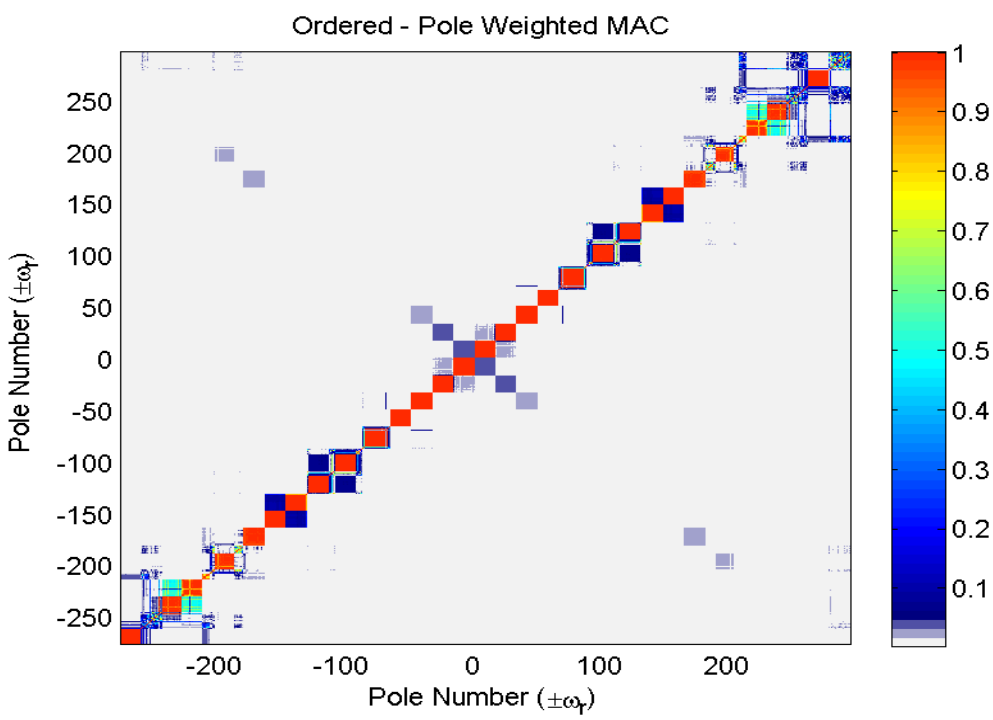

Figure 9. Auto-MAC Color Plot of Tenth Order Pole Weighted Vectors, No Threshold

The size of the red squares in Figure 9 (and in the following Figure 10) represents the number of vectors in each cluster of poles found anywhere in the consistency diagram.

4.6 Step 6: Remove Pole Weighted Vectors Below Threshold

Retain all Auto-MAC values that have a pole weighted MAC value above a threshold, 0.8 works well for most cases. All values below the threshold are set to 0.0.

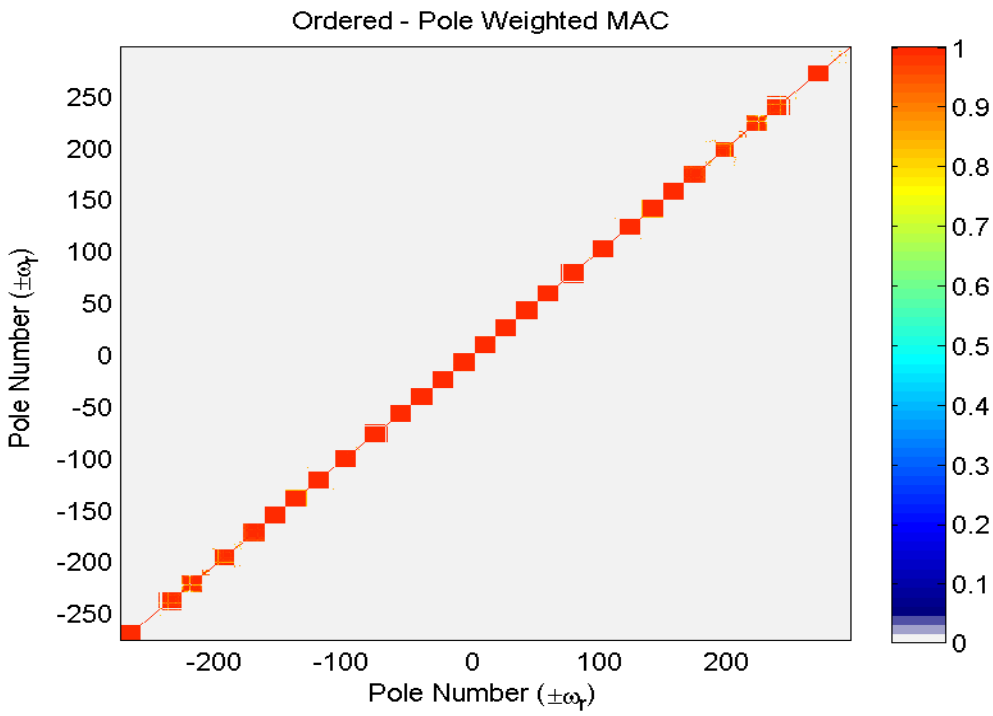

Figure 10. Auto-MAC Color Plot of Tenth Order Pole Weighted Vectors, Above Threshold

4.7 Step 7: Identify and Retain Consistent Pole Weighted Vector Clusters

Identify vector clusters from this pole weighted MAC diagram that represent the same pole weighted vector. This is done by taking the singular value decomposition (SVD) of the pole weighted MAC matrix. The number of significant singular values for this MAC matrix represents the number of significant pole clusters in the pole weighted vector matrix and the value of each significant singular value represents the size of the cluster since the vectors are unitary. Note that the singular value is nominally the square of the number of vectors in the cluster and will likely be different, mode by mode. Recognize also that Figure 10, as a thesholded Auto-MAC plot, will be largely zero and have significant magnitude (above the threshold) only for the pole weighted vectors that represent a cluster.

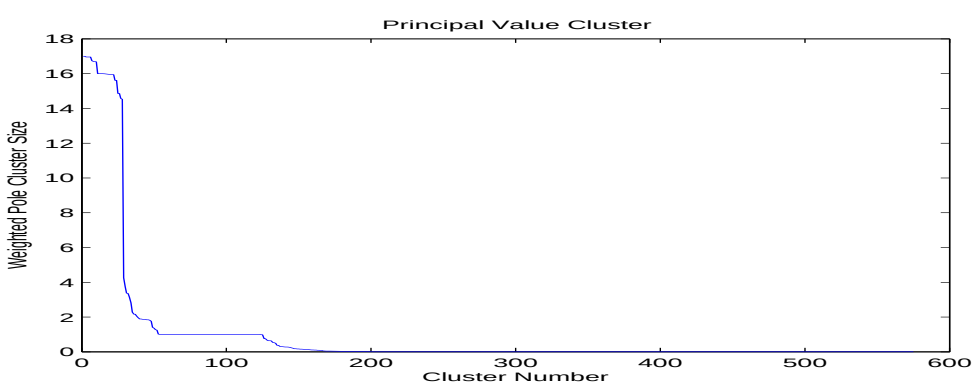

Figure 11. Principal Values of Clusters of Pole Weighted Vectors

Figure 11 then is a plot of the scaled singular values for Figure 10. Each significant singular value as determined by a minimum cluster size threshold (typically 4) represents a cluster of pole weighted vectors with an equivalent spatio-temporal characteristic. The location of the corresponding pole weighted vectors in the pole weighted vector matrix (index) is found

from the associated left singular vector. The singular vector associated with these significant singular values is nominally a logical index vector (when scaled by the square root of the singular value) and is used to determine the location in the original pole weighted vector matrix for those pole weighted vectors that are identified as being in a consistent cluster. This is accomplished by multiplying the left singular vector by the square root of the singular value and retaining all positions (indexes) above a threshold (typically 0.9). The positions of the non-zero elements in this vector are the indexes into the pole weighted vector matrix for all vectors belonging to a single cluster.

4.8 Step 8: Evaluate Pole Cluster

Effectively, Step 7 yields, cluster by cluster, the pole weighted vectors that will be used to identify a single consistent spatio-temporal set of data representing a single complex modal frequency, modal vector, and modal scaling. For each identified pole weighted vector cluster, a singular value decomposition (SVD) is performed on the identified set of pole weighted vectors. Note that the number of pole weighted vectors included in each cluster will in general not be the same. The significant left singular vector is the dominant (average) pole weighted vector. Use the zeroth order portion of this dominant vector to identify the modal vector and the relationship between the zeroth order and the first order portions of the dominant vector to identify the complex modal frequency. Complete the solution for modal scaling by using any MIMO process of your choice. A complete discussion of the choices can be found in the literature [48].

4.9 Step 9: Complete Statistical Evaluation for Each Pole Cluster

Since each cluster in Step 8 involves a number of estimates of complex modal frequency, modal vector, and modal scaling, computing statistics on the variation in the cluster is a natural part of this autonomous modal parameter estimation process. Tables 1 to 3 are examples of statistics that can be easily computed based upon these data clusters. Complete details about these statistics, their rationale and how each is computed can be found another paper [59].

Naturally, the statistics in these tables give the possibility, with experience, of developing an automatic threshold on any number of physical or statistical values to exclude solutions which are deemed physically unrealistic or statistically unacceptable. At this time, this is left to user interaction at the completion of the CSSAMI processing.

Index	Sample Size (N)	Frequency (Hz)	Damping (Hz)	Frequency (Hz) (Mean)	Damping (Hz) (Mean)	Std. Dev. (Hz)
8	16	362.564	-3.1666	362.564	-3.1670	0.1677
19	16	363.860	-3.5650	363.860	-3.5650	0.1251
10	16	557.055	-2.8966	557.055	-2.8966	0.0795
12	16	761.224	-5.2229	761.224	-5.2229	0.1008
2	17	764.190	-2.5371	764.190	-2.5371	0.1529
16	16	1222.980	-4.0883	1222.980	-4.0883	0.0271
13	16	1224.055	-3.9540	1224.055	-3.9540	0.0419
4	17	1328.036	-6.6495	1328.036	-6.6507	0.0907
17	16	1328.803	-5.4761	1328.803	-5.4768	0.1871
6	17	2019.269	-8.2512	2019.271	-8.2511	0.2543
23	15	2023.802	-7.5597	2023.801	-7.5598	0.0642
26	10	2321.862	-3.9074	2321.860	-3.9070	0.1690
28	8	2322.335	-3.8356	2322.335	-3.8367	0.2244
22	15	2337.895	-3.7967	2337.895	-3.7971	0.2287

TABLE 1. Modal Frequency Statistics

Index	N	Frequency (Hz)	$NMVR_1$	$NMVR_2$	$NSVR_1$	$NSVR_2$
8	16	362.564	0.0046	0.0730	0.0046	0.0730
19	16	363.860	0.0034	0.0543	0.0034	0.0543
10	16	557.055	0.0001	0.0014	0.0001	0.0014
12	16	761.224	0.0007	0.0092	0.0007	0.0093
2	17	764.190	0.0020	0.0312	0.0020	0.0312
16	16	1222.980	0.0017	0.0213	0.0017	0.0213
13	16	1224.055	0.0013	0.0185	0.0013	0.0184
4	17	1328.036	0.0042	0.0718	0.0042	0.0717
17	16	1328.803	0.0022	0.0342	0.0022	0.0342
6	17	2019.269	0.0054	0.0763	0.0054	0.0763
23	15	2023.802	0.0047	0.0537	0.0047	0.0537
26	10	2321.862	0.0162	0.1617	0.0162	0.1617
28	8	2322.335	0.0161	0.1279	0.0161	0.1278
22	15	2337.895	0.0020	0.0238	0.0020	0.0238

TABLE 2. Modal Vector Statistics

Index	N	Frequency (Hz)	ModalA (State Vector)				ModalA (Mean from cluster)				
			real	imaginary	magnitude	phase	real	imaginary	magnitude	phase	Std. Dev.
8	16	362.564	3.9663e+002	-5.7201e+003	5.7338e+003	-86.03	3.9574e+002	-5.7195e+003	5.7332e+003	-86.04	1.3866e+002
19	16	363.860	-4.8353e+001	-4.8716e+003	4.8719e+003	-90.57	4.5311e+001	-4.9271e+003	4.9273e+003	-89.47	3.1271e+002
10	16	557.055	7.3166e+001	-9.7205e+003	9.7207e+003	-89.57	8.4818e+001	-9.7107e+003	9.7111e+003	-89.50	1.2295e+002
12	16	761.224	9.6911e+001	-1.8433e+004	1.8434e+004	-89.70	9.5750e+001	-1.8433e+004	1.8433e+004	-89.70	2.1414e+002
2	17	764.190	-3.1250e+002	-7.0511e+003	7.0580e+003	-92.54	-3.0277e+002	-7.0524e+003	7.0589e+003	-92.46	3.1198e+002
16	16	1222.980	1.1082e+003	-1.7259e+004	1.7295e+004	-86.33	1.1154e+003	-1.7253e+004	1.7289e+004	-86.30	5.3096e+002
13	16	1224.055	-4.6394e+003	-3.0840e+004	3.1187e+004	-98.55	-4.6441e+003	-3.0074e+004	3.0431e+004	-98.78	1.6271e+003
4	17	1328.036	-4.8959e+002	-1.5810e+004	1.5818e+004	-91.77	-4.7949e+002	-1.5803e+004	1.5810e+004	-91.74	8.5516e+002
17	16	1328.803	1.5461e+003	-1.4661e+004	1.4742e+004	-83.98	1.5462e+003	-1.4659e+004	1.4741e+004	-83.98	2.9335e+002
6	17	2019.269	8.1564e+002	-1.3301e+004	1.3326e+004	-86.49	7.3874e+002	-1.3345e+004	1.3365e+004	-86.83	6.6888e+002
23	15	2023.802	4.2669e+003	-6.3248e+004	6.3392e+004	-86.14	4.1880e+003	-6.3244e+004	6.3382e+004	-86.21	4.7741e+003
26	10	2321.862	1.6011e+003	-2.9188e+004	2.9232e+004	-86.86	8.1739e+002	-3.0454e+004	3.0465e+004	-88.46	5.2745e+003
28	8	2322.335	2.8552e+003	-3.6135e+004	3.6247e+004	-85.48	1.7616e+003	-3.7289e+004	3.7331e+004	-87.30	6.3617e+003
22	15	2337.895	-1.3348e+003	-5.0815e+004	5.0833e+004	-91.50	-1.3637e+003	-5.0848e+004	5.0867e+004	-91.54	2.6546e+003

TABLE 3. Modal Scaling Statistics

The philosophy involved to this point is to allow all possible poles and vectors that demonstrate some degree of physical and statistical significance, in terms of the pole weighted vector consistency, to be retained by the procedure. The rationale is that it will be easier for a user to interact with this set of final estimates and intelligently remove poor estimates based upon the statistical or other information rather than to have to add estimates to the final identified set of modal parameters. Since each mode cluster provides a set of answers, the statistics that can be computed is one easy way to identify poorly estimated modal parameters. These poor estimates will occur based upon excessive noise in/on the measured data, violation of the assumption of the modal parameter estimation algorithms, or inadequate spatial data in terms of input-output sensor locations. Ultimately, the limitations of information theory will dictate whether a satisfactory set of modal parameters will be obtained.

4.10 Step 10: Assess Quality of Results (User Interaction)

Once the final set of modal parameters, along with their associated statistics, is obtained, quality can be assessed by many methods that are currently available. The most common example is to perform comparisons between the original measurements and measurements synthesized from the modal parameters. Another common example is to look at physical characteristics of the identified parameters such as reasonableness of frequency and damping values, normal mode characteristics in the modal vectors, and appropriate magnitude and phasing in the modal scaling. Other evaluations that may be helpful can be mean phase correlation (MPC) on the vectors, an Auto-MAC looking for agreement between the modal vectors from conjugate poles or any other method available.

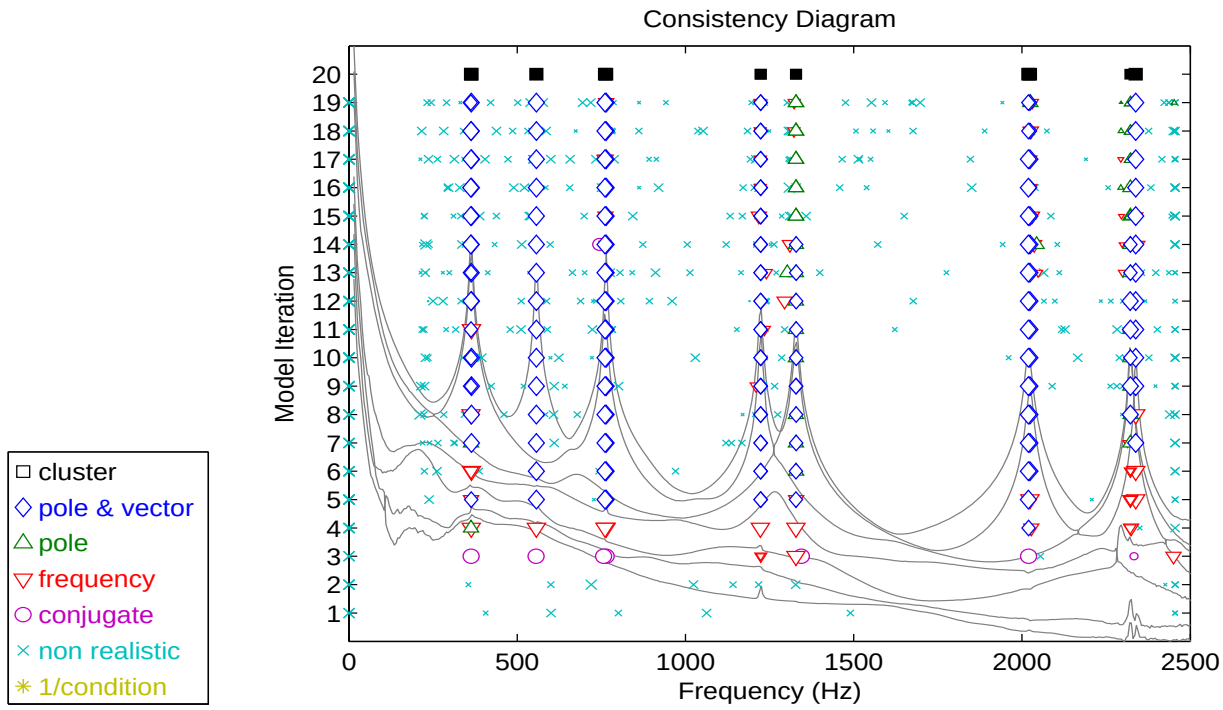

Figure 12. Consistency Diagram with Final Autonomous Estimates

Figure 12 shows the location of the final estimates on a consistency plot and Figure 13 shows their location a pole surface consistency plot. The black squares represent a final solution from the autonomous procedure.

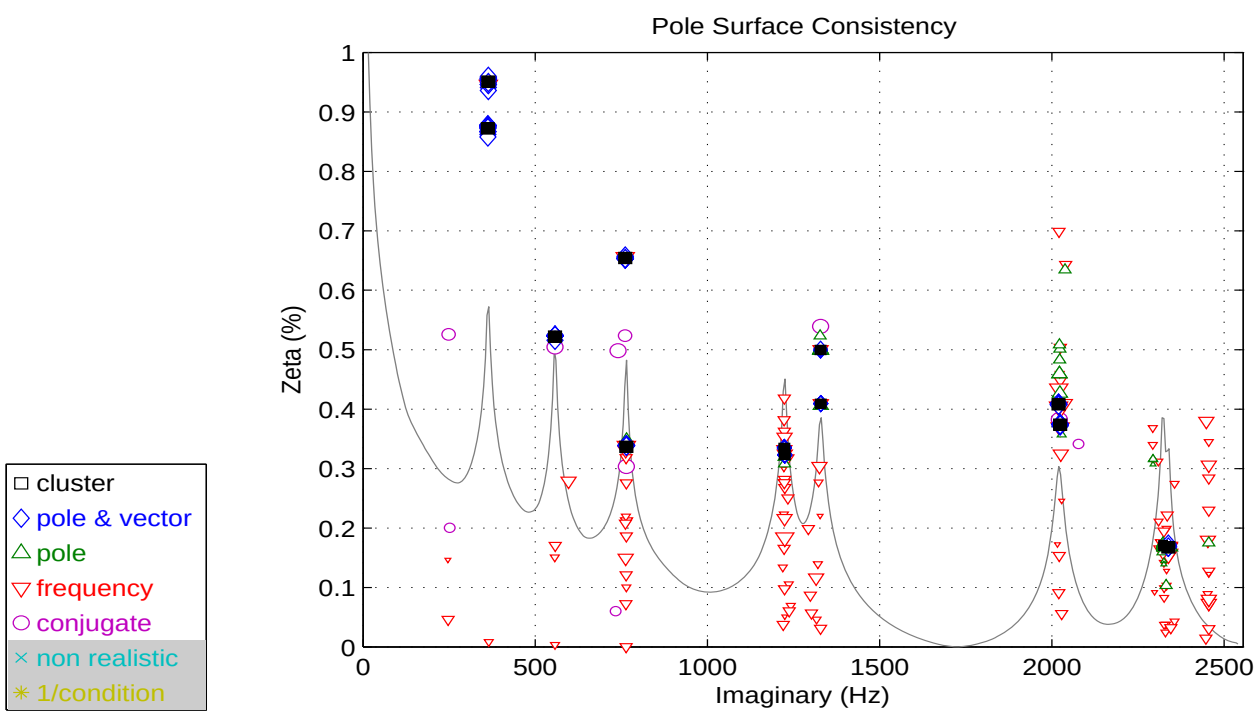

Figure 13. Pole Surface Consistency Clusters with Final Estimates

Figure 14 shows the Auto-MAC comparison of the resulting vectors with their conjugates. The results highlight the quality of the fit.

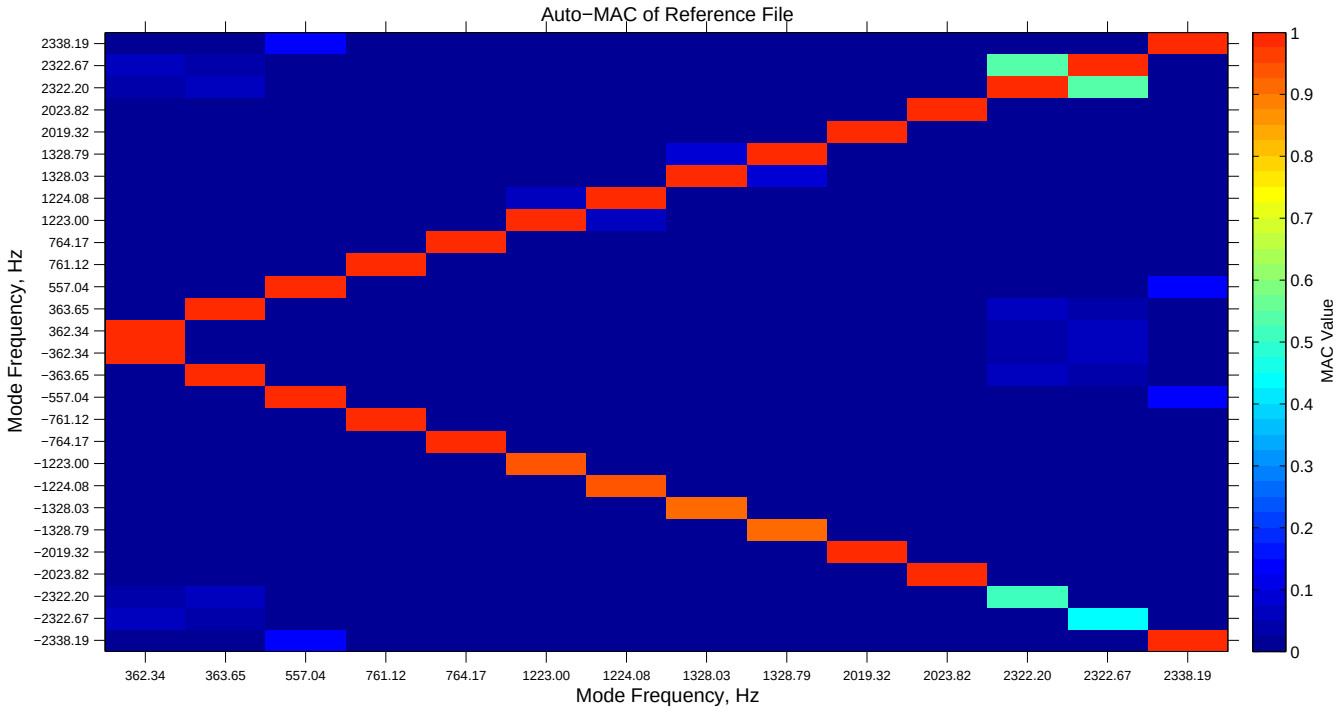

Figure 14. Auto-MAC of Final Autonomous Estimates - Conjugate Poles

5. User Control of Autonomous Process

In the course of this paper, a number of references to the algorithmic control parameters has been made. In this section, a brief discussion of these parameters and their application to the various steps of the procedure is presented. To enhance both visual impression and comprehesion, and because the procedure interacts with other display/consistency parameters, some relevant example dialog GUI(s) are shown in Figures 15 and 16.

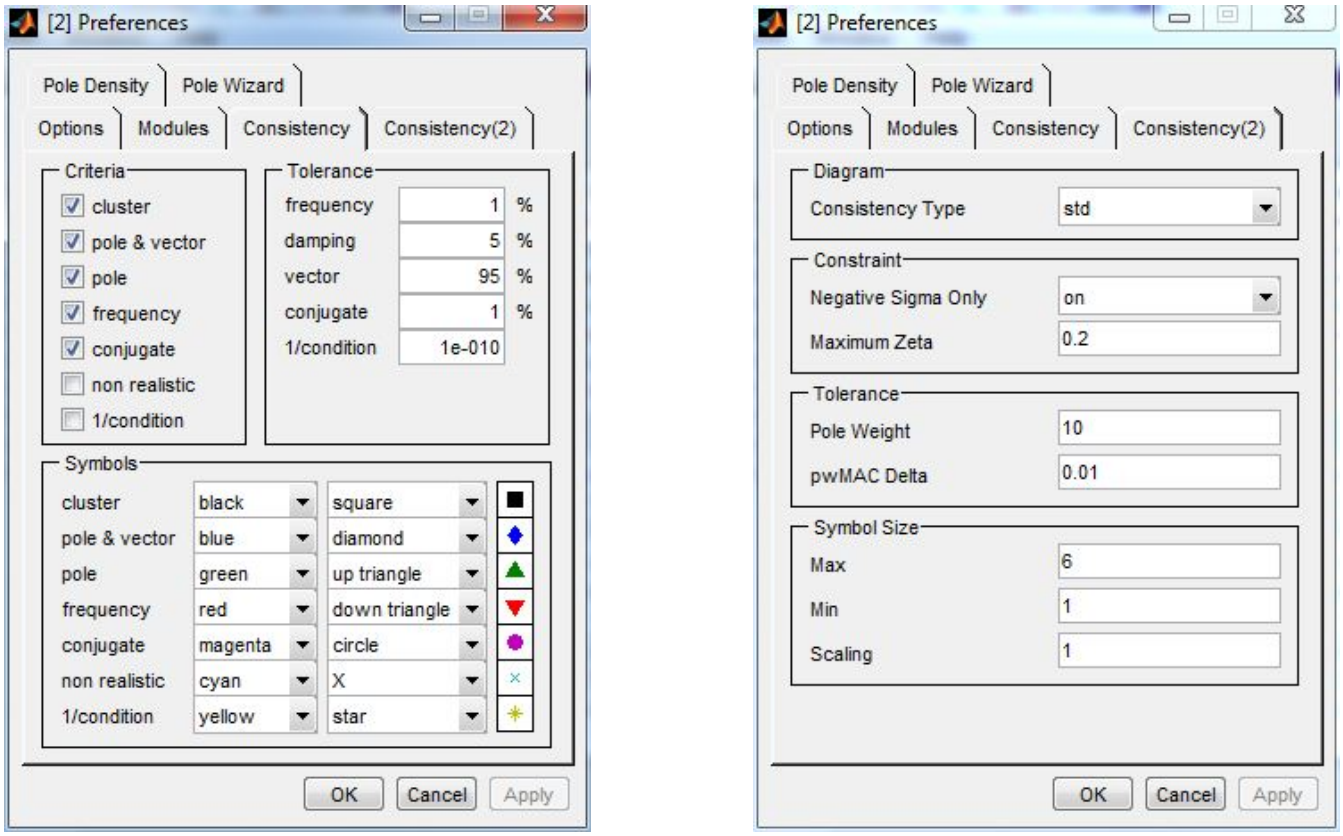

Figure 15. Pole Consistency Preferences GUI

- Consistency: Criteria - This group of parameters determine which set of consistency condition symbols will be displayed on the consistency and pole surface consistency diagrams. This affect procedure Step 1.
- Consistency: Tolerance - This set of parameters determine the criteria for which a pole (and vector) are considered consistent from one model iteration to another. This affects procedure Step 1.
- Consistency(2): Constraint - This pair of parameters is used to define the value of damping (real part of the pole) which is considered realistic. This affects procedure Step 1.
- Consistency(2): Tolerance: Pole Weight - This parameter determines the power of λ or Z used in the evaluation of the pole weighted vector. This affects procedure Step 4.

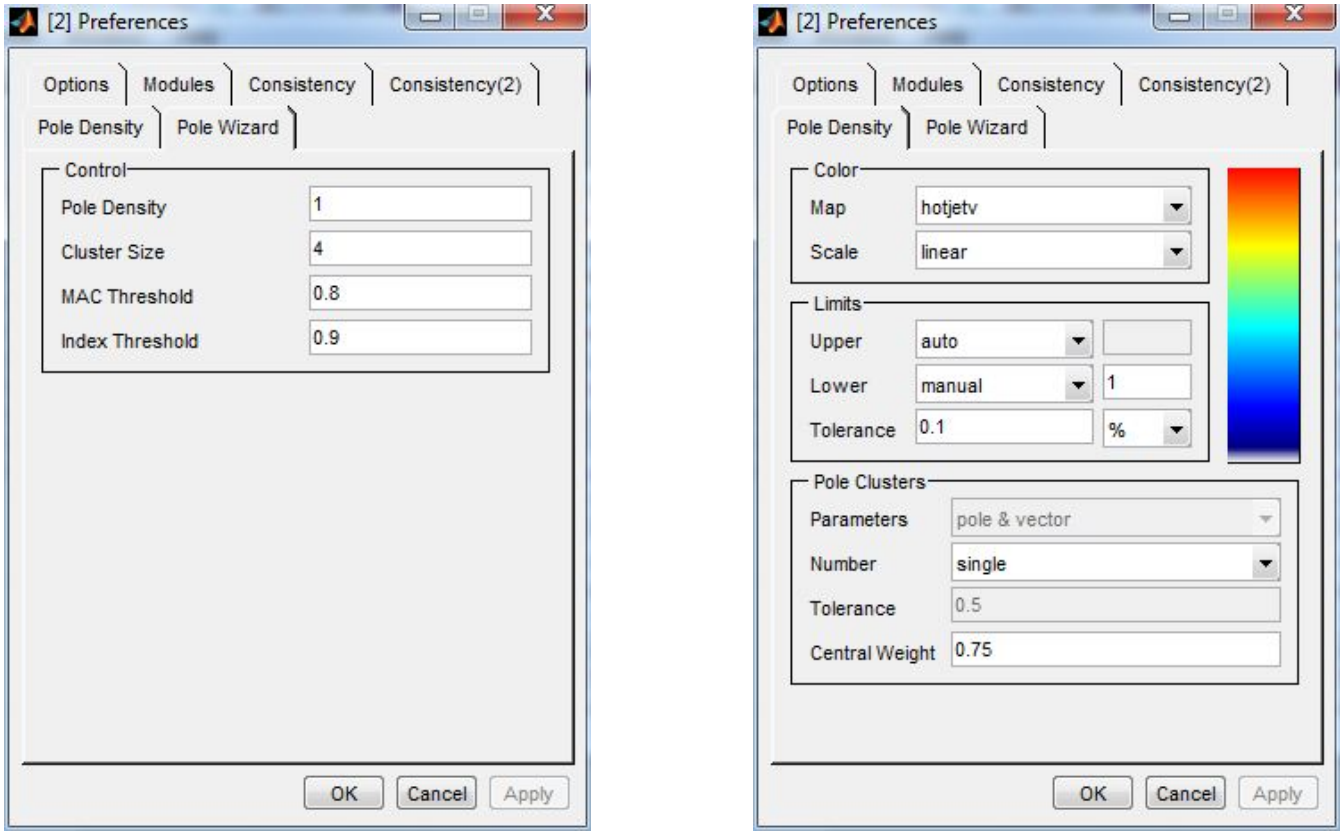

Figure 16. Pole Wizard and Density Preferences GUI

- Pole Wizard: Pole Density - This parameter selects for inclusion in the processing all displayed poles whose density is greater than or equal this value. As shown in the GUI, the value of one uses all displayed poles without limiting the number of poles evaluated. This affects procedure Step 3.
- Pole Wizard: Cluster Size - This parameter determines the minimum number of poles considered to represent an acceptable consistent spatio-temporal cluster. This affects procedure Step 7.
- Pole Wizard: MAC Threshold - This parameter determines the minimum MAC value which is considered significant in the determination of valid vector clusters. This affects procedure Step 6.
- Pole Wizard: Index Threshold - This parameter determines the minimum value of the scaled singular vector at which it is still considered a valid logical index. This affects procedure Step 7.
- Pole Density: Limits: Tolerance - This parameter determines the complex frequency radius around a pole in which any other pole is within this radius is considered to be part its cluster. The pole density is the number of poles located within the circle. This affects procedure Step 3.

6. Summary and Future Work

In this paper, a general autonomous modal parameter identification procedure has been presented along with a review of the history and theory applicable to the development of the autonomous procedures. This autonomous parameter estimation method utilizes the concept of evaluating statistically significant pole weighted vectors among thousands of possible solutions as the primary approach to identifying modal parameters. This involves the correlation of pole weighted vectors from multiple (more than two) sets of solutions involving either long or short basis vectors. This technique has been shown to be general and applicable to most standard commercially available algorithms.

With the advent of more computationally powerful computers and sufficient memory, it has become practical to evaluate sets of solutions involving thousands of modal parameter estimates and to extract the common information from those sets. The

autonomous procedure gives very acceptable results, in some cases superior results, in a fraction of the time required for an experienced user to get the same result.

It is important, however, to reiterate that the use of autonomous procedures or *wizard* tools by users with limited experience is probably not yet appropriate. Such tools are most appropriately used by analysts with the experience to accurately judge the quality of the parameter solutions identified.

Future work on this method planned by the authors include: combining the solutions from different UMPA methods (PTD plus RFP-Z plus ERA, for example), further evaluating the sensitivity of control parameters and thresholds, enhancing the statistical descriptors for each estimated set of modal parameters, and investigating more application cases such as purely academic, theoretical examples as well as difficult sets of measured, real-world data cases, including automotive, flight flutter and operating data applications.

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