

Review of Spatial Domain Modal Parameter Estimation Procedures and Testing Methods.

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Abstract

As part of the IMAC Technology Center a exhibition of continuously scanning laser, digital image correlation and photogrammetry modal testing will be presented. This paper is the “*kick off paper*” in a special session on these optical testing methods which offer the possibility of very high spatial resolution with 3 to 6 degrees of freedom at each spatial point and which have tremendously reduced requirements for sensor arrays and cabling systems used in a large modal test. The testing procedures for conducting test using these technologies will have to be adjusted to accommodate the scanning procedures. Many of the testing procedures of the past may be very well suited for these scanning methods and need to be reviewed. In particular methods based upon using spatial information in the initial stages of extracting modal parameters. This paper presents a review of spatial domain modal testing parameter estimation and testing procedures. A spatial domain testing method developed in the late 80’s and early 90’s (Spatial Sine Testing -- SST) will be review in more detail since it appears to be well suited for these optical scanning methods.

1 Acronyms

3D	Three Dimensional	SDRC	Structural Dynamics Research Corporation
ADC	Analog Digital Converter	SDOF	Single Degree of Freedom
ARMA	Auto Regressive Moving Average	SST	Spatial Sine Testing
CMIF	Complex Mode Indicator Function	SVD	Singular Value Decomposition
EMIF	Enhanced Mode Indicator Fuction	UC	University of Cincinnati
DSP	Digital Signal Processing	UMPA	Unified Matrix Polynomial Algorithm
GVT	Ground Vibration Test	UIF	Unit Impulse Function
FRF	Frequency Response Function		
HP	Hewlett Packard		
IRF	Impulse Response Function		
LSCE	Least Squares Complex Exponential		
MAC	Modal Assurance Criteria		
MIMO	Multiple-Input Multiple-Output		
MRIT	Multiple Reference Impact Testing		
NASA	National Aeronautics and Space Administration		
PTD	Polyreference Time Domain		
SDRL	Structural Dynamics Research Laboratory		

2 Introduction

Modal parameters are the complex valued modal frequencies and scaled modal vectors of the equations of motion of a linear system. Since the modal parameters are often found as the solutions of an eigenvalue problem, the modal parameters are frequently referred to as the eigenvalues and scaled eigenvectors of the equations of motion of a linear system. The equations describe the input-output characteristics of the system. The discussion in this paper is limited to structural systems, but in general it applies to any linear electro-vibro-acoustical system. The eigenvalues (temporal) and eigenvectors (spatial) are a set of characteristic functions which can be used to synthesize the relationships between the inputs and outputs of the system by superposition. The spatial information describes the location and orientation of the inputs and outputs of the system, while the temporal information describes how the responses change as a function of time and/or frequency due to the inputs. This paper presents a historical review of spatial domain procedures used to enhance the modal parameter process when measuring frequency response functions (FRFs). In particular, the history of how spatial information has played an increasingly important role in modal testing is tracked from the early days of experimental modal analysis, beginning with analog methods in the 1950s.

3 Background

In the late 1950s and early 1960s, two independent groups of researchers were actively trying to solve two very important self-excitation vibration problems. The first problem was a self-excited aerodynamic flutter problem, primarily involving aerospace structures. The second problem was a machine tool self-excited chatter problem. Both problems involved an unstable process where the deformation of the structure and the forcing function acting on the structure are coupled in a manner such that the forcing functions are influenced by the deformation. This process can be modeled as a closed-loop feedback process where the system's structural characteristics and the forcing mechanism can be used in the feedback process to predict the system's stability. Historically, it was the work stimulated by these two groups that led to the modern state-of-the-art practices in experimental structural dynamics and, to a lesser extent, analytical structural dynamics.

The machine tool problem involved only the relative force and motion between the tool and the work piece at a single point, whereas the aerodynamic flutter problem involved the 3D aerodynamic flow over the aircraft involving hundreds of points. Due to the nature of the two problems, spatial domain (eigenvector-mode shape) information was directly measured for the flutter problem, while temporal domain (directional *frequency response function* (FRF)) information was directly measured for the machine tool chatter problem.

In the late 1950s and early 1960s, digital computers with significant computational power were limited to large companies and government laboratories. Large mainframe computers were required for the early developments in the lumped mass modeling and finite element model fields. Even this type of computing capability was not sufficient for implementing any type of practical *digital signal processing* (DSP). Signal processing was limited to sinusoidal signals using analog filtering methods. It was the advent of the *fast Fourier transform* (FFT) algorithm in the mid 1960s that made DSP practical.

The two groups addressing the aircraft flutter and machine tool chatter took two different approaches for generating experimental models of the aircraft and machine tool structures. In performing a *ground vibration test* (GVT) of the aircraft, a force appropriation (normal mode) testing procedure was used. In this testing method, a tuned, multiple exciter sine testing method was used. Initially, a slow sweep sine or step sine technique was used to identify the approximate natural frequencies of the aircraft. Then at each identified

frequency, the frequency and forcing vector from an array of exciters strategically placed on the aircraft were perturbed until a normal mode was excited. When a normal mode is excited the phase of every point on the structure (normally 100 or more points) is in phase or 180 degrees out of phase with a set of reference points on the structure. The forcing pattern suppresses adjacent modes and removes the effects of non-proportional damping in the structure. In order to ensure a single mode is excited, the forcing pattern is removed and the free decay of the vibration is observed at selected points on the structure. If the free decay consists of a single frequency, the mode is properly tuned. The normal mode vector is obtained by simply measuring the magnitude and phase at every point on the structure. In the 1960s, tuning the modes was more of an art than a science and a skilled test team was a requirement, particularly for systems with closely coupled modes, heavy damping, and nonlinearities. However, when done correctly, this method gave superior results, except in situations where repeated modes existed.

For the machine tool chatter case, it was only necessary to measure the directional FRF, or the FRF between the relative response between the tool and work piece response in the chip thickness direction and the cutting force. From this measured FRF, the chatter could be related to various modes of vibrations in the machine tool. In order to understand the characteristics of the structural modes, a modal survey was conducted on the machine tool using a small hydraulic actuator and/or electro-mechanical exciter. Vibration modes that contributed to chatter instabilities could be identified from the directional FRF. In order to modify the machine, either an impedance model or a modal model needed to be developed for the machine tool. The measurement research effort of the late 1960s and early 1970s focused on generating an impedance model or modal model from a set of measured FRFs.

The two approaches outlined above were differentiated by the following major considerations:

- The flutter group measured eigenvectors directly (spatial domain method), while the machine tool group measured FRFs (temporal information).
- To be practical, the normal mode method required 4 to 12 exciters and a large number of response sensors. The FRF method initially used a two-channel system (single input-single response), with a roving response channel.
- When done correctly in the 1960's, the normal mode method gave excellent results for aircraft structures.

However, over the next 40 years, the estimation of FRF s followed by modal parameter estimation based upon FRF models became the dominant methodology, with the spatial information being more thoroughly incorporated into the process. Today, all FRF based methods that involve multiple input and multiple output data (often referred to as polyreference or MIMO methods) can be thought of as spatial domain methods.

For this paper, pure spatial domain methods are defined as those methods where spatial information (vectors) are estimated first, followed by estimation of the modal frequencies, damping and vector scaling parameters. While FRF methods have increasingly utilized more and more spatial information, very few of the FRF based methods would be considered pure spatial domain methods. Most of these MIMO FRF based methods work well using data acquired with laser scanning and/or photogrammetry testing methods that generate large amounts of spatial information. One of the methodologies is *spatial sine testing* (SST), developed in the late 1980s and early 1990s. SST is reviewed in some detail since it appears to be more compatible with the above-mentioned data acquisition methods.

4 Theoretical Considerations

In this section a very quick overview of the basic equations relevant to identifying the concepts of spatial domain versus temporal domain will be summarized. The development of these equations can be found in detail in the references.

The equations of motion for a simple discrete mechanical system with viscous damping can be written as follows:

$$[M]\{\ddot{x}(t)\} + [C]\{\dot{x}(t)\} + [K]\{x(t)\} = \{f(t)\} \quad [1]$$

where:

$[M]$	Mass matrix
$[K]$	Stiffness matrix
$[C]$	Viscous damping matrix
$\{x(t)\}$	Displacement vector
$\{f(t)\}$	External force vector
t	Time

Fourier transforming Equation (1), ignoring initial conditions:

$$[-\omega^2 [M] + j\omega [C] + [K]]\{X(\omega)\} = \{F(\omega)\} \quad [2]$$

Solving for $\{X(\omega)\}$,

$$\{X(\omega)\} = [-\omega^2 [M] + j\omega [C] + [K]]^{-1} \{F(\omega)\} \quad [3]$$

where ω = Frequency

The FRF matrix is:

$$[H(\omega)] = [-\omega^2 [M] + j\omega [C] + [K]]^{-1} \quad [4]$$

The FRF matrix expressed in terms of the eigenvalues and eigenvectors of the equations of motion in Equation (1) can be written as:

$$[H(\omega)] = [\Psi] \left[\frac{1}{j\omega - \lambda_r} \right] [L]^T \quad \text{where } \left[\frac{1}{j\omega - \lambda_r} \right] \text{ is a diagonal matrix} \quad [5]$$

Where $[\Psi]$ = matrix of eigenvectors – rth column is eigenvector for rth eigenvalue

$[L]^T$ = matrix of scaled eigenvectors – r^{th} row is transpose of r^{th} eigenvector scaled by modal scale factor for r^{th} mode – often referred to as modal participation matrix

λ_r = eigenvalue of r^{th} mode

The inverse Fourier transform of Equation (5), ignoring initial conditions, can be written as follows:

$$[h(t)] = [\Psi][e^{-\lambda_r}][L]^T \quad \text{where } [e^{-\lambda_r}] \text{ is a diagonal matrix} \quad [6]$$

Where $h(t)$ = Unit Impulse Response Function
 t = Time

Matrices $[\Psi]$ and $[L]$ are functions of position/direction and contain only spatial information, whereas $\left[\frac{1}{j\omega - \lambda_r}\right]$ and/or $[e^{-\lambda_r}]$ is only a function of time or frequency (temporal information). Those methods estimating the eigenvectors in the first step of the parameter identification process are referred to as *spatial domain* methods, while those estimating the eigenvalues and/of both eigenvalues and eigenvectors in the first step are referred to as *temporal domain* methods.

4.1 Characteristic Space

From a conceptual viewpoint, the *frequency response function* (FRF) and/or *unit impulse response function* (IRF) solutions of the equations of motion (Equation [5]) can be visualized as occupying a volume where the coordinate axes are defined in terms of the three sets of characteristics. Two axes of the conceptual volume correspond to spatial information, and the third axis is temporal information. The spatial coordinates are in terms of the input and response points of the system and the eigenvectors are the characteristic functions which occupy this space. The temporal axis is either time or frequency, depending upon the domain of the measurements and eigenvalues defining this space. These three axes define a volume which is referred to as the *Characteristic Space* (see Figure 1).

It is possible to estimate the total space by experimentally measuring a subspace which samples all three characteristics. Experimentally any point in this space can be measured. The particular subspace which is measured and the weighting of the data within the subspace are the main differences between the various modal parameter estimation procedures which have been developed historically.

Information parallel to one of the axes consists of a solution composed of the superposition of the characteristics defined by that axis. The other two characteristics determine the scaling of each term in the superposition. In Figure 2, the line represents a subspace which is typical of an FRF or IRF measurement. In Figure 3, the plane is the subspace of all measured response data due to a single input excitation point. Figure 4 is the response due to a multiple excitation

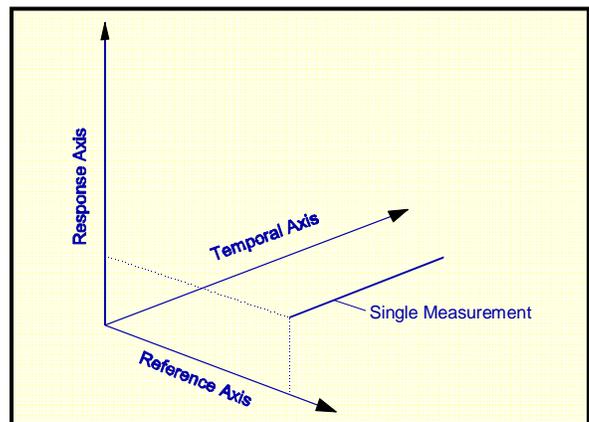


Figure 1 -- Characteristic Space

case. Figures 2-4 represent measurements containing temporal information.

It should be noted that theoretically if all modes are excited, the data from any plane intersecting the characteristic space can estimate all the modal parameters provided there are no repeated roots. It requires two planes for a multiplicity two root or three for multiplicity three, etc.

For *spatial domain* methods, the planes of subspaces are parallel to the plane of the response and reference axes.

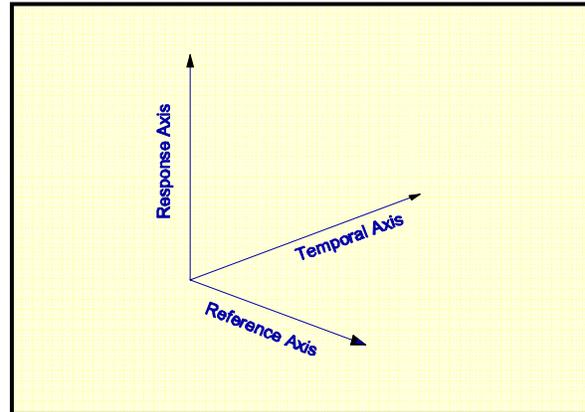


Figure 2 – Typical Frequency Response or Unit Impulse Response.

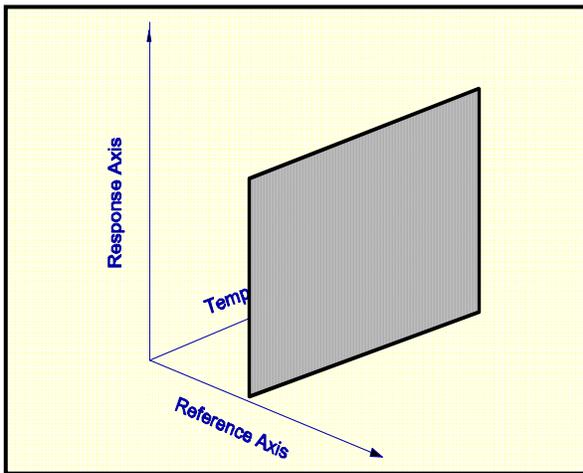


Figure 3 – Data subspace single input

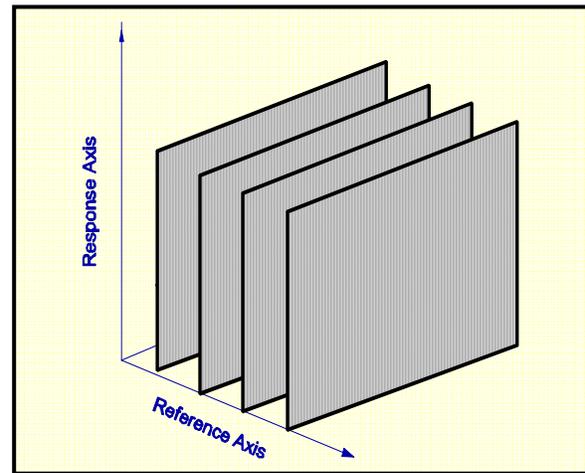


Figure 4 – Data subspace multiple input

5 Historical Review of Spatial Domain Methods

The purpose of this review is to revisit spatial domain modal testing procedures to determine if they can be adapted to work with current optical scanning methods, image correlation and/or photogrammetry) that can extract modal properties of optically observable test objects. Optical methods are capable of measuring forced modes of vibration with high spatial resolution as a function of frequency. This section presents a brief review of signal processing and modal parameter identification, with a more detailed review of relevant spatial domain developments in each decade from the 1960s to the present.

5.1 1960s

Force appropriation, normal mode sinusoidal testing was the original spatial domain testing method, first developed in the 1950s. The approximate natural frequencies were located in an initial step sine test by observing the frequencies where the amplitudes of the responses peaked. In the early 1960s, the development

of the tracking filter and the resulting *transfer function analyzer* (TFA) made swept sine testing practical and made it significantly easier to locate the resonant frequencies. At each of the resonant frequencies, the frequencies and forcing vector were perturbed to identify the normal mode. The forcing vector removed the non-proportional damping (phase of every observed response point is 0 or 180 degrees with respect to forcing vector), resulting in a normal mode. If the exciter locations have been poorly chosen, the required phase criteria may not be able to be met. An updated version of this testing procedure is still used today for large aerospace structures by a number of test organizations.

Historically, the phase was obtained by observing Lissajous patterns on a large array of oscilloscopes. Today, computers monitor the modal vectors and control the forcing vector.

During the 1960s, there were significant improvements in making FRF measurements, using TFAs in the early 1960s and Fourier Analyzers in the late 60s. The Fourier Analyzer system allowed FRFs to be measured with a wide variety of excitation conditions, including transient testing methods (primarily impact testing).

Normal mode testing was not practical for most testing organizations due to the high capital equipment investment (100s of sensors and 4 to 12 exciters). It also required a very skilled test team to tune the modes. As a result, the major research efforts of the 1960s were in the development of digital signal processing and parameter estimations methods for extracting modal information from FRF measurements. Once fast Fourier transform methods became available in test equipment, this method began to lose favor for the above reasons, particularly as modal parameter estimation algorithms capable of extracting the system's modal frequency, damping, scaling and vectors from measured FRFs began to be developed in the 1970s. The preference for an FRF based methodology was also being influenced by newly developed impedance modeling methods where FRF measurements could be used directly to develop structural system models.

5.2 1970s

As mentioned in the previous section, using parameter estimation to extract modal parameters from FRF measurements was one of the goals of modal testing. The initial attempts at formulating *multiple degree of freedom* (MDOF) modal parameter estimation algorithms involved organizing the unknowns in a non-linear equation form. The numerical methods developed to solve this type of non-linear mathematical problem in the 1960s and 1970s were numerically unstable. They worked well on systems where the modes were uncoupled, but when the modes were coupled the methods had problems with stability. As a result, SDOF approximations (quadrature, peak picking, circle fitting, etc.) were mainly used to obtain estimates of modal parameters from FRF measurements.

In the early 1970s, the *complex exponential algorithm* (CEA)^[1] was evaluated as a modal parameter estimation method for extracting modal parameters. CEA was a time domain method and it could extract modal parameters from a measured unit *impulse response function* (IRF). The CEA method fit estimates of the IRFs such that the fitted data matched the measured data with very small errors. For a short time, the goal of estimating modal parameters had been reached! Unfortunately, when FRF data with high modal coupling were processed with CEA, the fitted data matched individual measurements nearly perfectly, but the eigenvalues changed significantly between measurements. The eigenvalues from different measurement points should be the same. As a result, it became clear at this point that it was necessary to fit more than one FRF at a time and to include more spatial information in the modal parameter procedures.

Two algorithms were developed in the mid 1970s that utilized a complete data set (hundreds of FRFs) taken from a single exciter location. The first method, developed in the mid 1970s, was Ibrahim's Time Domain (ITD) method.^[2] ITD could take a complete data set for a single reference and formulate an eigenvalues solution which computed both the eigenvalues and eigenvectors. The limitation of ITD in the early 1970s was that it was a computationally intensive algorithm and required a large computer system.

The second method was the Least Square Complex Exponential (LSCE) Algorithm ^[3], which computed a least-squares estimate of the system eigenvalues in the first step. It used the estimated eigenvalues in the second step to estimate the residues by using a frequency domain algorithm to fit the individual FRFs. The major advantage of the LSCE Algorithm was that it could run on the mini-computer built into the commercially available HP Fourier Analyzer system. Both methods utilized spatial information and emphasized the importance of having consistent data sets, or data sets where the eigenvalues and eigenvectors were the same in each measurement.

5.2.1 Little MAC (Modal Assurance Criteria)

In the early 1970s, the main research effort was in digital signal processing using Fourier analysis, including excitation methods, sampling, windowing, coherence, etc. The main effort was to make the best possible measurements. One of the fallouts of this effort was the development of a purely spatial domain modal parameter estimation procedure (Little MAC) which could be used with the two-channel analyzers of the early 1970s.

Little MAC ^[4-6] was based upon the idea that the system eigenvector was a system characteristic and not a function of excitation location or type. This method measured the FRF between a reference accelerometer mounted on the test object and a roving accelerometer mounted at points of interest on the test object. An excitation signal was applied at random points on the structure and the responses were measured at the reference location and the roving point of interest. An FRF and a coherence function were calculated between the reference response and the roving point response. The FRF measures the ratio of the modal coefficients between the reference point and the roving point. The coherence function peaks at the modal frequencies of the system. The reason for this is that, in the frequency band around a modal frequency, the modal vector for that mode dominates the response and the ratio of responses in the frequency band remains fairly constant for all the randomly applied inputs.

The estimation of Little MAC is essentially the coherence function calculated between the roving accelerometer and reference accelerometer. If the reference sensor is located at a point of maximum response for a given mode, then the H_1 FRF estimator gives better results for that mode. The reason is that there is less noise on the input sensor due to the contribution of other modes.

The responses for the two points of interest are proportional to the forcing function acting on the structure during the averaging period, thus the forcing function's influence will cancel out in the ratio. The FRF for the response at point p due to the input at point q is shown below for the frequency band where mode k dominates.

$$H_{pq}(\omega) = \sum_{k=1}^N \left(\frac{Q_k \psi_{pk} \psi_{qk}}{j\omega - \lambda_k} + \frac{Q_k^* \psi_{pk}^* \psi_{qk}^*}{j\omega - \lambda_k^*} \right) \approx \frac{Q_k \psi_{pk} \psi_{qk}}{j\omega - \lambda_k} \quad [7]$$

The ratio of the response between point p and point r due to a force acting at point q becomes:

$$R_{pq} \approx \frac{A_{pq}}{A_{rq}} = \frac{H_{pq} F_q}{H_{rq} F_q} = \frac{\psi_{pk}}{\psi_{rk}} \quad [8]$$

- Where
- Q_k = Modal Scale Factor for eigenvalue λ_k
 - ψ_{pk} = Modal Coefficient at point p for eigenvalue λ_k
 - A_{pq} = Response at point p due to force at point q

Note that the response ratio is independent of input location so randomizing the input location allows Little MAC to indicate where the response ratio is consistent as the input location is varied (Little MAC will not be consistent off resonance). This test method was used on several small test articles by mounting a reference sensor and a roving sensor, and then rapidly impacting at a large number of random points for each roved point. The data was processed by treating it as a random data set, applying a Hanning window, and taking a significant number of averages. Excellent mode shape data was obtained, even when the roving sensor caused noticeable frequency shifts.

A great deal of effort was required to randomly impact the structure in order to make a measurement for a single point. The effort had to be repeated for each roved point. Because of this effort, it was more of a laboratory curiosity and never became a standard test method. If an array of sensors was mounted, which today is a fairly standard testing practice, the method would become more practical. It may be a method which should be re-examined in terms of the current interest in response only modal analysis efforts.

Little MAC created several important spin-offs. MAC, or what was referred to as *Big MAC*, was developed at the same time in which the same computational algorithm was used to correlate two different modal vectors. If the MAC value is equal to one, the modes are exactly correlated. As the MAC value decreases, the correlation decreases accordingly. The *Modal Assurance Criteria* (MAC) is now in common usage in most commercial software and is used for correlating modal data and as a validation tool in many modern modal testing programs^[5-7].

5.2.2 Multiple-Input, Multiple-Output (MIMO) FRF Testing

To take advantage of increased spatial data, MIMO FRF testing methodologies were examined in the late 1970s. The initial work was done using a 4-channel system, and later with an 8-channel system and several test structures^[5, 8-11]. It was clear when using the ITD or the LSCE methods that the data set had to be very consistent; in other words, the modal parameters had to be consistent (unchanging over the whole test) for the complete data set being processed. As a result, significant effort was spent on signal processing, eliminating measurement errors due to sampling, randomizing inputs to minimize non-linear distortion error, etc. It was desirable that all the inputs and response points are measured simultaneously and that the data be throughput with at least a few real-time channels to insure that excitation signals were satisfactory and that FRF and coherence measurements were acceptable. The throughput data allowed the data to be post-processed off-line with different signal processing (different blocksize, zoom bands, windows, averages, etc). Unfortunately, only a few companies or organizations had the measurement resources to conduct this type of test.

5.3 1980s

There were a large number of developments in the 1980s that enabled the measurement, processing and analysis of spatial information. In 1981 Boeing Aircraft set the stage for the 1980s decade by conducting the first large full scale test using MIMO FRF testing and modal parameter estimation structures^[8]. Figure 5 shows the test setup. Boeing mounted several hundred sensors on the aircraft and ADC throughput the data to a large removal disk pack which could be transferred to a second system for data processing while the testing system took data on another configuration of the aircraft. They processed just enough data in real time to determine if the test was proceeding as planned. This was the ultimate test system envisioned in the 1970s. The ADC data could be reprocessed with different signal processing methods to examine the data in more detail if there were concerns during the post-processing. The results of this test confirmed that taking a complete simultaneous data set improved the processing of the data and resulted in better modal parameters. The data for this test was processed with LSCE to obtain the estimated eigenvalues using the data from multiple references in a least square sense. However, LSCE is not a multiple-reference parameter estimation



Figure 5-- Boeing Test Setup 1981

algorithm. It could not uncouple repeated roots or very closely coupled modes. In the next section, the breakthrough algorithm (Polyreference Time Domain (PTD) ^[12-13]) that allowed repeated modes to be estimated is reviewed.

This test clearly demonstrated the advantage of incorporating more spatial information into the modal testing process. Unfortunately, the cost of this type of system was still limited it to a large organization where the test was a requirement and the company could make the investment in the required equipment. The positive aspect was that it was used to prompt companies (Hewlett Packard and PCB) that supplied modal testing equipment to make an investment in the hardware development that could support multiple channels at a more reasonable per channel cost.

5.3.1 Polyreference Time Domain

The Polyreference Time Domain (PTD) method ^[12-13] was a multiple reference version of LSCE and, as the first multiple reference algorithm, was a revolutionary breakthrough development. The additional spatial information made it possible to separate very closely coupled modes. In fact, repeated modes could be calculated. For this type of algorithm, MIMO FRF testing and simultaneous measurements were important. PTD made the development of large channel count modal systems a critical requirement. The method was introduced by the Structural Dynamics Research Corporation (SDRC) at about the same time as the Boeing test. This was additional incentive for HP and PCB to commercialize multiple channel acquisition and inexpensive sensors.

5.3.2 Structcel Development

In the early 1980s, it was becoming obvious that it was necessary to improve multichannel data acquisition, signal processing, and parameter estimation. Hewlett Packard (HP), the major supplier of Fourier Analyzer systems, was interested in developing a new high channel count acquisition system, but the cost was too expensive per channel for the existing marketplace. The downfall was the combined cost of the computer, data acquisition system and sensor system. At that time, there was little possibility that a significant reduction in cost could be achieved for the computer/data acquisition system which was their part of the combined system. A major effort was launched to locate a sensor manufacturer who would be willing to develop a low-cost sensor (target of less than 15 to 20 US dollars). Sensor systems at that time were 500 to 1000 dollars per

channel and even with expanded channel count, the sensor manufacturer could not make a business case to recover the investment. As a result, HP started to develop an inexpensive sensor.

However, in 1984, several graduate students at the UC-SDRL were given a thesis project to develop a low-cost sensor system. Fortunately, one of the student's father and uncle were key partners in PCB Piezotronics, a sensor company. The student convinced the company to commercialize the sensor system. PCB did not meet the 15 dollar price but sold the sensor for 29 dollars in quantity without a cable and 49 dollars with the cable. This was a major breakthrough which helped HP to justify their new multiple channel system. The sensor had good dynamic range, barcode compatibility, snap-in mounting base, insulation displacement connectors, ribbon cables, simple cable management, etc. The major disadvantage was a significant calibration drift due to changes in humidity. The sensor needed to be calibrated frequently, resulting in the development of a 120-channel array calibrator. As an interesting aside, once a company bought several hundred channels of data acquisition, then during the next purchasing cycle many customers updated their sensors to higher quality sensors from PCB. This helped PCB ultimately justify the initial investment into what appeared to be a questionable business venture.

Obviously, the Structcel System ^[14-15] was a very important development and spurred the further development of modal testing procedures. These procedures generated a more consistent measurement database and led to improved identification procedures.

5.3.3 HP 3565 Multiple Channel Data Acquisition System

Hewlett Packard introduced the HP 3565 multiple channel data acquisition system (Polygon) at the 1986 IMAC conference. The Structcel was also introduced at the 1986 IMAC. The Polygon system and the Structcel sensor system triggered even more research into better multiple channel parameter estimation algorithms, test management software, and more vendors introducing competing products during the late 1980s to early 1990s.

5.3.4 Modal Parameter Estimation - Unification Project

In the early 1980s, with the development of time domain (ITD, LSCE and then PTD) algorithms followed by several frequency domain algorithms (RFP, Orthogonal Polynomial Methods and PFD), it became very difficult to explain each method based upon the very different developments of the original authors. There was a need to unify the development of the varying algorithms so that the advantages, disadvantages, similarities and differences were easier to understand. Three events occurred that spurred the development of modal parameter estimation in this direction. First, NASA expressed interest in unifying the modal parameter identification methods. The major problem was that each method had generated a following with users having strong convictions, supporting their method of choice. NASA wanted an unbiased appraisal of the methods, and hopefully some way to unify them. NASA gave a grant to the UC-SDRL to try to unify the modal parameter estimation methods. This started a decade long effort to develop an understanding of the interrelations between the various algorithms ^[16].

Second, at this same time, there was considerable interest in developing a frequency domain, direct parameter estimation algorithm to estimate the mass, stiffness and damping matrices directly from the measured forces and response vectors. The student working on this problem struggled with obtaining a stable numerical solution due to poor conditioning of the frequency domain data. In the mid 1980s, based upon this work, a unifying approach for the time domain methods began to emerge ^[17-18].

Finally, in the mid 1980's, UC-SDRL was in the process of documenting the important modal parameter estimation methods for the US Air Force. This required UC-SDRL to develop a complete understanding of the formulation of these methods. After some time and study, it was discovered that all the methods could be reformulated from a common starting point, in both the time and generalized frequency domains ^[21].

Depending upon the number of observed degrees-of-freedom, the equations of motion could be reformulated by algebraic manipulation into a polynomial with an order different from the second-order equations of motion. Initially, the generalized formulation was referred to as the Auto Regressive Moving Average (ARMA) method, which would include cases where the input forces were not observed. There were small groups of researchers working on machine tools, controls, and civil engineering infrastructure who were using the ARMA methods. These groups were working fairly independently in the 1980s from the modal analysis community. As a result, there were several reviewers who suggested a name other than ARMA, since an ARMA model was purely a time domain method and had moving average terms. All of the modal parameter identification techniques reviewed used either free decays or measured FRFs which only required autoregressive terms. Since the common starting point formulation was a polynomial with matrix coefficients, this was later renamed the Unified Matrix Polynomial Algorithm (UMPA) ^[22-24].

UMPA Models -- General Formulation for the kth equation;

$$\sum_{i=0}^n [\alpha_i] \{x(t_{k+i})\} = \sum_{j=0}^m [\beta_j] \{f(t_{k+j})\} \quad \text{Time Domain} \quad [9]$$

$$\sum_{i=0}^n (s_k)^i [\alpha_i] \{X(s_k)\} = \sum_{j=0}^m (s_k)^j [\beta_j] \{F(s_k)\} \quad \text{Generalized Frequency Domain} \quad [10]$$

All of the commercial modal parameter algorithms can be reformulated starting with an UMPA model.

This combined work resulted in a set of course notes that has been used since the late 1980s in the System Dynamics Analysis (SDA) II course at the University of Cincinnati, and has been used as reference material in industrial and government agency courses. A copy of these notes can be found on the UC-SDRL website. [http:// www.sdrl.uc.edu](http://www.sdrl.uc.edu).

5.3.5 Sine Testing Methods

In the mid 80's several new sine testing methods were developed which combined many of the features of *normal mode* testing methods and FRF testing methods. In 1986 SDRC developed the *MultiPhase – Step Sine Approach* ^[25-27] which varied the phase of the forcing vector in a monophasic fashion. This estimated FRF's from the data by using multiple sweeps with difference phases between the exciters (for example, one sweep with the exciters in phase and with the exciters out of phase). This allowed FRFs, mode indicator functions and advanced parameter estimations methods to be used to estimate the modal parameters. The phasing of the shakers could be optimized to extract selected target modes. The method was also compatible with standard force appropriation (normal modes) techniques. The *multiple variable mode indicator function* (MvMIF) could be used to aid in the tuning of individual modes since FRF data was available. At the same time, the University of Leuven developed a step sine testing method ^[28-29] and continuing improvement to the *normal*

mode testing methods were being made at other national testing laboratories, primarily in France and Germany. Many of these techniques have been continuously updated and improved over the years and are currently used primarily in the aerospace industry.

5.3.6 Complex Mode Indicator Function (CMIF)

In the mid to late 1980s, a Complex Mode Indicator Function (CMIF) was developed which was based upon the Singular Value Decomposition (SVD) of the FRF matrix (See Figure 6), or the imaginary component (quadrature response) of the FRF matrix^[30-32]. Initially, this was used as an indicator function for determining how many modes were significant in a frequency band. Later it was used as a modal parameter estimation algorithm. It is purely a spatial domain method. The mode shapes are estimated directly from the SVD. The CMIF modal vectors were used as estimates of the eigenvectors. With a large number of references, this method works well on systems with high modal coupling. It works by measuring the quadrature response operating mode shapes as a function of frequency for all the references (excitation points). At each frequency, an SVD is performed to determine the dominant quadrature mode shape at that frequency. This enhanced quadrature mode shape is used as a modal filter to determine an Enhanced Frequency Response Function (EFRF)^[33]. The EFRF enhances the mode of interest and a simple SDOF method can be used estimate the eigenvalue and modal scale factor. This is a technique that can easily be adapted to optical scanning methods. CMIF will be discussed in more detail in the Spatial Sine Testing (SST) and Multiple Reference Impact Testing (MRIT) discussions later.

5.3.7 Perturbed Boundary Condition Testing

In the Perturbed Boundary Condition (PBC) testing procedure^[34-36], the boundary conditions are drastically perturbed and the system retested to generate a larger database which can be used to update a finite element model or to generate a more complete experimental model of the system. The simplest modification is to add rigid masses at selected boundary points. A finite element model is built for each configuration. It is a simple modification to the finite element model to add a rigid mass. If all the transducers are installed on the

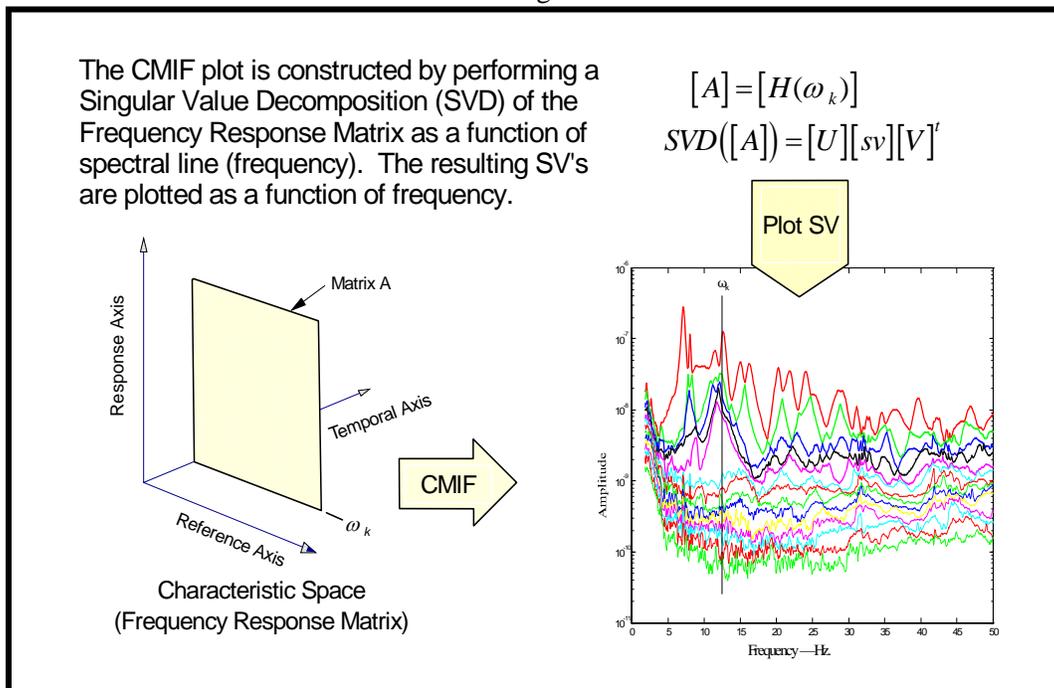


Figure 6 Schematic of CMIF Process

structure it is simple to test a second configuration, with the only difference being that sensors are added to monitor the added mass. Testing multiple configurations generates a larger correlation database.

In the spatial sine testing methods, the perturbed boundary adds passive forces which can be incorporated into the UMPA parameter estimation process to enhance the experimental results.

In 1987, NASA performed a modal test on the Astro-1 Satellite that demonstrated many of the advancements that were made during the 1980s. The test was a MIMO test with random excitation, 500 channels of Structcel sensors, and PBCs with mass-loaded boundary conditions. CMIF was used as a mode indicator function and to obtain initial estimates of the eigenvectors. See Figure 7 for the test setup of the Astro 1 modal test. The large yellow masses were used to mass-load the boundary points.

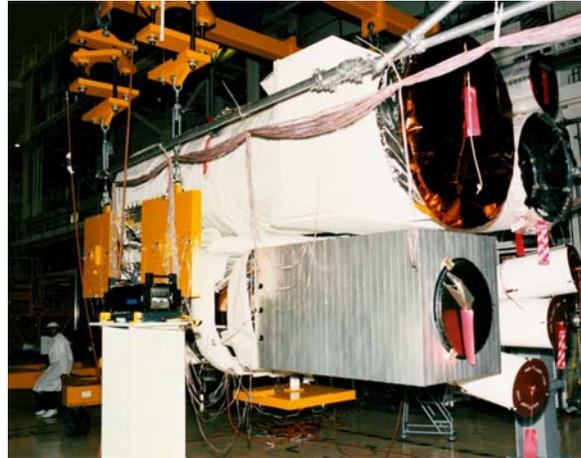


Figure 7 -- Astro-1 Test Setup 1987

5.4 1990s

In the late 1980s and early 1990s, several spatial domain methods using concepts incorporated into the CMIF development were refined for use in Spatial Sine Testing (SST) and Multiple Reference Impact Testing (MRIT).

5.4.1 Spatial Sine Testing (SST)

During the 1970's and the early 1980's, there was a significant effort to improve the *force appropriation, normal mode* testing procedures based upon sinusoidal excitation^[37-41]. Parameter estimation concepts and mode indicator functions were being incorporated into the process to make it easier to tune the modes.

In the late 1980's, an SST system^[42-45] was developed as a very inexpensive, high channel count, sine testing system. The system used an IBM 286 PC as the computer and could in real time measure and process 512 input-output channels. The cost was to be 250 to 300 dollars per channel for sensor and acquisition. It used the PCB Structcel and Data Harvester. The basic Data Harvester was the ICP signal conditioning for the Structcel, but in the late 1980s, it was modified to include a switched capacitor, low pass filter which could be used for anti-aliasing protection. A DSP board was used in the IBM PC to perform the digital signal processing which included providing the sinusoidal excitation signal, sampling the input and response data, and estimating the Fourier coefficient at the frequency of the sine wave using a four-point DFT.

In the early 1990s, the signal processing was moved inside the Data Harvester. Each 8-channel signal conditioning board in the Data Harvester was modified with an 8-channel multiplexed ADC and a switched capacitor, low pass filter to provide the signal conditioning and digital signal processing for the SST system. A 16-channel sine source board was added to output 16 synchronized sine waves with both controllable amplitude and phase and a clock signal to control the sampling of the ADCs. A 512-channel system consisted of four Data Harvesters daisy-chained together and controlled by the IBM PC through an HPIB interface.

The signal processing in the updated Data Harvester consisted of measuring one period of the sine wave sampled with either four or eight points. The switched capacitor filter acted as a tracking anti-aliasing filter which passed only the first harmonic. A number of cycles of the digitized sine wave were time averaged to

reduce the signal noise in the Data Harvester. A 4-point or 8-point single frequency DFT was performed on the averaged time data, and the real and imaginary parts of the first harmonic were transferred to the computer. The resulting data consisted of a forced mode of vibration due to the excitation force vector. In reality, the system acted as a 512 channel co-quad analyzer. It measured the real and imaginary components from all sensors (response and force) relative to the master sinusoidal drive signal. The computer was then free to control the sine excitation and sweep rate and to estimate the modal parameters.

The testing sequence was to step up or down in frequency, accumulating forced modes of vibration. The sweep was a continuous process but the sweep rate, step size, and the number of forced responses in the band were adjusted depending upon the number of modal frequencies identified in a narrow frequency band. Initially, a frequency domain UMPA model was used with *singular value decomposition* (SVD) as a mode indicator function to identify the number of modes in a narrow frequency band. The SVD was also used to determine the rank of the response vector matrix and to condense the response vectors.

This testing procedure worked well, but the computer in 1988-90 was not fast enough to keep up in real time. So, instead of the SVD, a recursive LU decomposition was used. This method allowed the forced response vectors to be added or subtracted to the response vector matrix without going through a complete decomposition each time. Periodically, a complete LU or SVD solution was required to keep the solution on track. Today, with significantly improved compute power a SVD condensation would be used.

A major difference of the SST modal parameter identification algorithm is that it uses directly the measured forced response vectors and forcing vectors, while most other modal parameter algorithms use FRFs or IRFs. It can also include passive forces in its forcing vector. A passive force is the force that is generated by an attached subsystem. An example of this is the masses attached to boundary points in the PBC testing described in the previous section.

The SST Algorithm is based upon using a first order, narrowband frequency domain UMPA model. The generic form for this first order model is:

$$\left[(j\omega) \begin{bmatrix} A_1 \\ A_0 \end{bmatrix} + \begin{bmatrix} A_0 \\ A_0 \end{bmatrix} \right] \{ \bar{X}(\omega) \}_k = \left[\sum_{m=0}^M (j\omega)^m \begin{bmatrix} B_m \end{bmatrix} \right] \{ \bar{F}(\omega) \}_k \quad [11]$$

The above equation is the generalized equation of motion for a system. $\{ \bar{F}(\omega) \}_k$ is the k^{th} forcing vector acting on the system and $\{ \bar{X}(\omega) \}_k$ is the k^{th} response vector due to both active and passive forces.

In the above equation the $[A]$ terms are the matrix coefficients of the polynomial that operates on the response vector $\{ \bar{X}(\omega) \}$, and the $[B]$ terms are the matrix coefficients of the polynomial that operates on the input vector $\{ \bar{F} \}$. The equation is true for every frequency and for every input and output vector. The roots of the response polynomials are the eigenvalues (poles) of the system and the roots of the input matrix are the zeros of the system. These zeros are used to describe the out of band residuals.

The size of the A matrix is determined by the number of modes in the narrow frequency band. For SST, the size of A is larger than the number of modes observed in the frequency band, and only a B_0 term is used. This allows the method to interpolate and find upcoming out-of-band modes during the sweep. This means that it anticipates upcoming modes in the sweep and the forcing function can be adjusted to enhance these modes and suppress modes already identified. The other adjustment that was made to improve the interpolation was to normalize either the A_1 or the A_0 matrix, depending upon the sweep direction.

This SST method was used in 1990 for a large channel count test performed on a SAAB 340 Aircraft. The test was a vibro-acoustic test to investigate the possibility of installing an array of mass dampers on the fuselage to reduce the internal sound pressure levels due to propeller noise.

The test was a 500-channel test with 280 Structcels mounted on the aircraft fuselage and 208 Acoustcels used to measure the acoustic modes in the aircraft cabin. Twelve small two-pound force Wilcoxon impedance heads were used to excite the aircraft. See Figures 8 and 9. The impedance heads were permanently mounted on the fuselage for the duration of the test. Later, a sailplane was tested with both the SST method and with a standard MIMO testing procedure and a non-linear space structure was tested at NASA Huntsville using the SST method.



Figure 8 – Saab Aircraft Spatial Sine Test

The SST system was developed primarily as an inexpensive large channel count system. It suddenly became clear in the mid 1990s that the technology revolution in the consumer marketplace was leading to tremendous improvements in computing, networking and signal processing. The implementation of delta sigma ADCs, used in CD, DVD and MP3 players, was going to drastically change the cost of multiple channel data acquisition systems. As a result, the commercialization of specialized sine testing was not continued. The rationale at the time was that an inexpensive broadband acquisition system would be available which could be used for both sine testing and MIMO broadband testing.

Ten to fifteen years later, the future is now. Utilizing new measurement techniques (continuously scanning lasers, digital image correlation and optical photogrammetry), measuring forced response modes in real time at thousands of points simultaneously is possible and becoming more practical each day. The SST should be one of the techniques investigated as a potential means of extracting model parameters from these measurements.

Commercial photogrammetry and laser systems are currently available which use sinusoidal excitation and aliasing to measure the 3D deformation patterns at thousands of points. These systems tremendously reduce the number of sensors, cabling and cable management problems. There is still going to be a need for ordinary sensors to monitor points that are not visible, so a hybrid testing system with a sparse sensors array may be necessary.

This sparse array of sensors using MIMO or MRIT testing methods could also be used to identify the frequency bands of interest for the SST method which would be used to refine estimates of the modal parameters in these bands and provide



Figure 9 – Structcel and Impedance Head Exciter Installation Spatial Sine Testing Setup

significantly enhanced spatial information.

For the testing with the SST method in the early 90's, the exciters were hard mounted to the structure and only the force acting normal to the surface was measured. The photogrammetry methods are capable of measuring the motion of the exciter body and if the end of the armature is visible it could also measure the motion of the armature motion. This could be used to estimate both the active force and the passive force generated by the exciter.

A new development that would be useful for the SST testing method would be a long stroke exciter which could be mounted on the test structure and its motion monitored. Two exciters of this type were used to excite several bridges and a large 26 story dormitory in the early nineties by the Department of Civil Engineering at the University of Cincinnati. Both exciters had significant mass in the armature which was used to generate an inertia force which excited the structure. The motion of the moving mass was measured to estimate the force. The passive forces due to the motion of the exciter body was not measured, however, this could have easily been done. In Figure 10, a photograph of both exciters is shown mounted on bridges.



Figure 10 – Electro-Mechanical Exciter - (Left) and Hydraulic (Right)

The electro-mechanical bridge exciter, shown in Figure 10, had a 32 in stroke and could generate a 1000 lbf RMS sinusoidal force down to one Hz. A scaled down version of this exciter could be developed for exciting large flexible structure.

The parameter estimation algorithm for the SST was the basis for the *enhanced mode indicator function* (EMIF) algorithm, developed in the early 1990s for the Multiple Reference Impact Testing (MRIT) which is discussed in the next section.

5.4.2 Multiple Reference Impact Testing (MRIT)

MRIT^[46] was a spin-off of the CMIF and SST efforts of the 1980s. MRIT uses CMIF and/or EMIF and measured FRFs from a multiple reference impact test to obtain modal parameters. The EMIF algorithm is the same frequency domain UMPA model that is used in the SST algorithm. The SST method uses a sinusoidal forcing vector exciting at multiple points on the structure, while MRIT uses impacts at multiple points. The two methods have several important points in common.

In common testing configurations, an array of 7 to 15 accelerometers are mounted on the structure and the impact hammer is roved. For larger structures, particularly those found in industrial or civil infrastructure testing, a small array of accelerometers are roved and 7 or more fixed points are impacted. If a large modal system is available, a large array of accelerometers is pre-mounted and impacts are applied at 7 or more fixed points.

In the early 1990s, the Civil Engineering Department at the University of Cincinnati was involved in testing a large number of bridges for the state of Ohio as part of a bridge health monitoring program. They were evaluating a number of modal testing procedures in order to develop a modal model of the system. The model was then used to predict bridge flexibility, which was one of the metrics they were using to evaluate the health of the bridge. The test methods included sine and random testing procedures using large hydraulic and electromechanical exciters, and impact testing. They were having difficulty getting a modal model from all of the methods using the latest state-of-the-art modal parameter identification procedures. They then pre-mounted a large array of seismic sensors on the bridge and impacted at 7 to 14 fixed points. Initially, the modes were processed with latest MIMO parameter estimation algorithms. The results from the various algorithms were evaluated by computing the deflection of the bridge due to loaded trucks parked on the bridge and comparing it to the measured deflection due to truck loads. The CMIF method yielded the best results^[48-51].

This impacting testing method also took the least amount of time, which was beneficial since they had to stop traffic to do the testing. A special MATLAB® program was developed for controlling the testing and a CMIF and EMIF plug was added to X-Modal program for processing the data.

The MRIT, CMIF and EMIF methods have been well-documented in theses^[30,32,46,47] and conference papers.

5.5 2000s

In the current decade, most of the improvement in testing has been a general evolutionally improvement in parameter algorithms with better and cheaper data acquisition - sensors systems. There has been a significant improvement in the usability of software in terms of extracting modal data. However, in terms of tradition testing methods there have been no significant improvements in the way spatial data is handled versus temporal data. Certainly, the understanding of the numerical issues involved with the matrix coefficient polynomial characteristic equation has been advance by work at the Vrije Universiteit Brussel, Belgium^[52-54]. This has led to understanding in the way to utilize stabilization (consistency) diagrams with all current modal parameter estimation methods.

One significant development of the 2000s that does involve spatial domain data is the continued development of response only modal parameter estimation methods. These methods attempt to determine the modal parameters by utilizing natural excitations that are not measured. Many of these methods utilize cross

correlation (covariance) methods in the time domain or cross spectra methods in the frequency domain. In both cases, the spatial characteristics of the data come from a significant number of response sensors. Since the force cannot be measured, assumptions concerning the force characteristics are utilized in order to process this data with many of the same algorithms used for conventional MIMO data used for experimental modal analysis. Without knowledge of the true inputs, the rank of the response data matrix, in specific frequency ranges of interest, may be much less than the number of sensors retained as the references in the data matrix. Just because there are a large number of response sensors (200) and a data matrix at each frequency can be formed on this basis (200 x 200) does not mean that the data matrix will have all of the possible modes of vibration in the frequency range of interest (modes may not be excited). The force assumptions of a large number of forces acting on the structure at a large number of points with a smooth frequency spectra is normally met in civil engineering structures but is not likely to be met by structures like an automobile.

6 Conclusions

The fundamental question is what methods are going to be used to handle the vast amounts of spatial data which can be measured using the newer continuously scanning, optical measurement techniques. These measurement techniques are currently becoming practical as long as the structure is optically observable. The nature of these measurement techniques, when observing a large number of locations, is currently suited for sine testing methods.

In this paper, the general overview of how spatial information has historically been incorporated into the modal parameter estimation process has been summarized. The force appropriation (normal mode) testing methods and the spatial sine testing methods are good candidate testing methods since they are pure sine testing methods and do not require FRF measurements or broadband FFT analysis. The traditional force appropriation methods require tuning and spatial sine testing requires putting force response data into a UMPA parameter estimation method. Both methods should work in a hybrid testing method where the sensor data is incorporated into the response vectors. The sensor data can also be used in a standard MIMO test to obtain frequency bands of interest and the optical data can improve the spatial resolution of the data in potentially all six degrees-of-freedom.

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