SYSTEM SUPPORT FOR SPATIAL SINE TESTING

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ABSTRACT-
With the success and acceptance of low cost motion sensing instrumentation, large scale modal testing has become feasible for small laboratories with fewer resources. Complementing the multichannel capabilities, sine testing methods, which take advantage of the increase in measurement points, also continue to grow in popularity. Savings in time, as well as increases in data consistency account for much of the methods' attractiveness. However, managing the large numbers of channels of data increase the common problems associated with configuration and calibration of the measurement system. Current research directed toward these "management" problems focus on fast, concurrent calibration and more efficient, inexpensive signal processing. The personal computer based signal processing under development utilizes a spatial sine testing technique which incorporates a large array of motion sensors, the discrete Fourier transform and advanced parameter identification algorithms.

tests, largely due to the availability of low cost motion sensors and signal conditioning. Placing renewed interest on the measurement process, sine testing has spurred new research not only in the instrumentation field but also in the signal processing and calibration areas.

No longer can the measurement process be thought of as single channels and vectors. Hundreds of channels and huge arrays are rapidly becoming standard in the field, creating new problems in instrumentation and data management. This "management" problem has become the focal point of a number of new research topics.

Providing an exceptionally simple approach to Fourier analysis, the four point discrete Fourier transform allows a desk top computer to perform all of the necessary mathematical calculations in a 6 input, 512 response sine tested modal survey. The algorithm requires that all data points be instrumented and that anti-alias filtering be applied to the time signals. Data is stored as spatial information (forced mode shapes) rather than as conventional frequency response functions (temporal information). This allows a number of parameter estimation algorithms to exercise temporal and/or reference weighting of the forced modes of

INTRODUCTION-
Sine testing has returned as a viable and efficient method for conducting modal
vibration while determining the modal characteristics of the test article. Due to the unique way the data is stored and manipulated, the testing technique has been dubbed the spatial sine testing.

Clearly, the requirement to instrument all data points generates the necessity for a large supply of motion sensors. With this instrumentation also comes the arduous jobs of mounting, wiring and identifying each channel. The Structcel system provides solutions to these problems. Incorporating many ergonomic or human engineering aspects, the system addresses each of the problems involved in managing very large numbers of channels of data. Coupled with low cost signal conditioning provided by the Data Harvester, the Structcel system also helps to automate the testing procedure while lowering the per channel cost.

Often considered troublesome and sometimes even overlooked, sensor calibration lies at the heart of every valid measurement. However, the calibration of hundreds of channels of instrumentation place phenomenal time and work loads on testing facilities. To help ease this burden, a fast, concurrent calibration method is presently under development along with the sine testing method. Utilizing a rigid body platform and a spatial domain algorithm that calculates the sensitivity of any one sensor against a reference and/or the average of all others, the method calibrates up to 256 at a time.

Figure 1 - Characteristic Space

SPATIAL DOMAIN AND THE SST METHOD-
Composing a unified thought process, the spatial domain describes a system frequency response and/or impulse function matrix in terms of the product of three characteristic functions; two complex spatial and one complex temporal. The spatial characteristics are a function of geometry while the temporal characteristic is a function of either time or frequency. Hence, any discrete point along a frequency response consists of a triplet grouping an input point, output point and discrete frequency.

Since attempting to visualize three dimensional complex space promotes migraine headaches, slices of the space parallel to the axes display a much more comforting picture (See fig. 1). Planes sliced parallel to the temporal axes correspond to individual frequency (or impulse) response functions. The frequency response functions consist of a summation of unit amplitude SDOF frequency response functions (temporal information), weighted by the two axes of the characteristic space.

Slices parallel to the response axis correspond to forced modes of vibration. These forced modes consist of a summation of the system eigenvectors weighted by the input characteristic and the temporal characteristic. Commonly referred to as modal participation factors, slices parallel to the input axis consist of a summation of the system eigenvectors weighted by the response characteristic and the temporal characteristic.

Managing spatial information rather than frequency responses, the spatial sine testing (SST) method stores data as forced mode shapes. The method excites the test structure at preselected but arbitrary frequencies. Thus the measured complex forced modes build a data base along with the frequency and measured forcing vectors. Important to note, the forcing vectors and frequency are chosen to build the data base, not to tune a mode as in force appropriation methods. As a result, the SST method allows the freedom to take unequal frequency steps.
Currently, several advanced multiple reference parameter identification algorithms, such as Polyreference Frequency Domain [ref. 1,2], Multiple Reference Orthogonal Polynomial [ref. 3-5] and Multi-MAC [ref. 6], process data of this form.

**SYSTEM CONCEPT:**

The measurement system focuses on one goal; to provide the best, most consistent modal data base in the most cost effective manner. Along with this goal, practicality and simplicity round out the major design criterion. The system under development divides logically into four groups: motion sensing, signal conditioning, force appropriation and signal processing/system control. (See fig. 2)

Having portions of the system already commercially available, the present design integrates prototype force actuation, signal processing and system control with commercial motion sensors and signal conditioning. Developed in conjunction with the University of Cincinnati, PCB Piezotronics designed their low cost motion sensing system for overall performance and cost effectiveness. Completed two years ago as phase I of the project, the sensors' and original signal conditioning's commercial success reinforce the value of the project design goals. Also commercially from PCB, the Data Harvester supplies low cost signal conditioning, amplification and filtering as phase II of the project. Much in the same manner, the present prototype developments for phase III involve continued review for performance and cost effectiveness.

The envisioned system utilizes a spatial sine testing method to acquire and process data in near real time. Reducing the seemingly insurmountable amounts data involved in a large multi-input/multi-output test, the discrete Fourier transform (DFT) provides a solution. Since the signal conditioning contains anti-alias filtering, the sinusoidal signals do not need to be over sampled to gain the desired amplitude accuracy. Instead, a simple four point DFT involving only two additions can be performed on the signals to produce the corresponding Fourier coefficients. This saves a tremendous amount of processing overhead when compared to FFT methods.

Supporting 256 or more response channels and up to 32 input channels, the testing system provides all rack mountable components [ref. 7]. Low cost, expandable signal conditioning/filtering mount in groups of 64 channels at a time while a separate utility box houses an inexpensive digital signal processing card, multiplexers and analog to digital converters. As the basis of the utility box, a simple, single board personal computer then provides system control, parameter estimation routines and animation software.

**COMPONENT DESCRIPTION:**

Sensors:

The structmoel measurement system provides an array of motion sensors at an affordable per channel cost [ref. 8-11]. Accordingly, all desired data points can be instrumented to provide a more comprehensive data base exhibiting better consistency and spacial definition. These qualities are especially important if modal modelling is to be attempted.

Light weight and high sensitivity, the sensors provide near ideal characteristics for large scale modal testing. The sensors output a nominal 1 volt/g sensitivity and provide a resolution of .001 g. By
instrumenting large numbers of data points (typically 64, 128 etc.), the three gram sensors provide a more comprehensive data base while adding only a minimal, constant mass distribution. Accordingly, the resultant data base displays better spacial resolution without the frequency shifts inherent to the "roving" accelerometer method. Leaving more time for data acquisition and analysis, the measurement system integrates many time saving features which help the test engineer to manage extremely large numbers of channels of data. Modular design, cut to length wiring, insulation displacement connectors and multiconductor bus cables all combine to speed the set up process. Sensor signals are routed to patch panels which consolidate the individual cables into multiconductor ribbon cables carrying signals on to the signal conditioning equipment. The orderly wiring scheme maintains the set up functionally intact for weeks allowing for as many modal surveys as needed.

Calibration and identification are also structured to expedite the testing process. In one step, a hand-held single frequency exciter simultaneously calibrates and identifies the channel with a constant one "g" signal. The signal also provides a continuity check for the wiring and connections. As an alternative to the hand-held calibration method, a second method is under development to suit organizations requiring laboratory precalibration. The fast, concurrent calibration under development will be discussed later in the calibration section.

Signal Conditioning:

Managing all channels in parallel, the Data Harvester provides power, amplification and anti-alias filtering. It eliminates tedious bookkeeping tasks and manual adjustments by interfacing multiple sensors to the computer. The system controller automatically sets and monitors all channels allowing the Data Harvester to provide high level, bandwidth limited, zero based sensor signal outputs.

Designed to compliment the Structcel system, the rack mountable unit consists of three parts: the power module, the processing module and signal conditioning cards. All parts reside in a standard 19 inch rack mountable case and can be accessed behind a smoked glass front panel.

Available in three versions, the power module supplies the signal conditioning cards with Structcel power, ICP power or both. The latter choice allows versatility for easy integration of conventional quartz ICP sensors into the system. Since the number of inputs is always less than 32, only one rack of signal conditioning requires the combined power. Hence, additional racks require only the low cost, dedicated Structcel power modules.

The processing module provides computer controlled access to the amplification/filtering process. Largely responsible for lowering the per channel cost, a powerful central processor allows up to 1024 channels of signal conditioning to be daisy chained to a single processing module.

Subsequently, the addition of extra signal conditioning racks ,with out the expense of additional processor modules, make system expansion very attractive. The processor module also facilitates fast, efficient autoranging and selectable filtering, which can be set and monitored either interactively or by computer control.

Each signal conditioning card accommodates four separate sensor channels. Containing the filtering and amplifying hardware, the cards plug into a mother board which accommodates up to 16 cards per rack. Filtering with a 48 dB/octave roll-off and amplifying with an adjustable gain of 8 - 50 dB in 4 dB increments provide adequate anti-alias protection and autoranging capabilities at a low per channel cost.

While conserving money, the Data Harvester also conserves space. A four rack (256 ch.) set-up takes up only 21 inches of vertical rack space. Adding a single board, rack mountable PC as the system controller brings the total height of the 256 channel sine testing system to about 30 inches.

Force appropriation:

Again focussing on functionality and cost, shakers constructed from ordinary speaker cones display promising results.
The speaker cones cost less than 10 dollars apiece and easily modify to make system compatible exciters. Multiple input locations distribute the input energy more evenly about the test structure. This in turn, produces better measurements at most locations.

A nylon rod threaded through the center of the cone acts as a point of attachment. Flexible in the lateral direction, the rod also acts as a stinger to transmit forces in only the axial direction. Trimming portions of the cones away creates a cradle from which the center hangs. This reduces the air damping and also helps constrain the motion to only the vertical direction. The input signals to the speakers originate from a DAC in the system controller. Shifted in both phase and amplitude, the single sine wave provides an uncoupled input for each exciter location. When used as exciters, the speaker cones exhibit an excellent frequency response range well beyond the practical 0 to 1000 Hz and a peak-to-peak stroke length of nearly 1/2 inch. The long stroke is especially important for providing sufficient input energy at very low frequencies.

Signal processing/system control:

Due to the unique measurement approach, the system control and signal processing overhead can be handled by a simple desk top computer. Restricting the input to be sinusoidal, the measurement system utilizes a four point DFT (4 pt. DFT) to generate the real and imaginary Fourier coefficients (See appendix II for explanation of the 4 pt DFT). This simplified ability to solve for the coefficients eliminates the need for a fast Fourier analyzer, which saves large dollar amounts and facilitates further investment in the front end capabilities of the measurement system. Once again, the increased capabilities lead to better spacial definition and a more consistent modal data base.

The system under development uses a modified IBM/AT as the system controller/processor. The modification involves a commercially available digital signal processing (DSP) board which contains analog to digital converters, memory and digital signal processing logic. The filtered and amplified signals pass through these components at near real time speed to generate and store the Fourier coefficients to memory. The board also contains a digital to analog converter which generates the sinusoidal driver signal for the reference shakers. Presently, researchers work to develop hardware that will accomplish the magnitude and phase shifting.

In addition to housing the DSP, the computer provides control to the Data Harvester. An RS-232 link sends and receives autoranging commands, filter settings, overload indications and inquiries as to setting values. Slightly more expensive to implement, an IEEE-488 connection communicates the same information.

Since the DSP provides immediate access to the Fourier coefficients, the IBM/AT also functions to solve for the system poles and damping, as well as provide animation software at record speed. Coupled with a high resolution display, the system displays animated mode shapes providing the user a view of relative amplitude and phase relationships. The computer also supports a number of calibration routines which provide visual monitoring of the calibration signals for any abnormalities.

CALIBRATION-

The standard method of system calibration involves a single frequency, one "g", hand-held calibrator. This method allows for end to end calibration, channel identification and a cable continuity check in a single step. Accordingly, this saves a great deal of time while simplifying the set-up process.

Unfortunately, this system approach to calibration can sometimes be unacceptable. Some testing groups require laboratory precalibration and/or strict traceability to the National Bureau of Standards (NBS). Another reason for possible dissatisfaction with the standard method of calibration is the physical requirement for a test engineer to be at each test point to hold, jumper and activate the calibrator. With test points two or three stories high on an aircraft tail,
the standard method may be unacceptable.

To handle problems such as the ones mentioned, a new fast, concurrent calibration (FCC) method is under development. Consisting of an instrumented rigid body mounting platform, the FCC method handles up to 256 Structcel calibrations at a time. The calibration system outputs a sensitivity for each sensors' main axis as well as the two orthogonal cross axes. To ensure adequate input in all three translational degrees of freedom, three electro-mechanical shakers provide simultaneous uncorrelated random input across the frequency range of interest.

An array of 256 motion sensors measure vertical motion while four reference triaxial accelerometers occupy the lower four corners of the platform. These reference accelerometers, which can be calibrated with NBS traceability, accurately measure the motion of the platform in all three translational directions. Utilizing the reference measurements and the measured geometry of the plate, the calibration algorithm uses the rigid body mode shapes of the platform to normalize the measurements. The algorithm then restricts the motion of the test sensors to agree with the rigid body results. To ensure validity, the rigid body assumption holds only in limited frequency range that is well below the first deformation mode of the platform. According to the rigid body assumptions, the sensitivity of any one test sensor can be computed in relation to the average of the reference accelerometers and/or relative to all mounted accelerometers. The use of averaging (both spatial and statistical) supplies a better representation of the motion of the platform. Two computational algorithms can be used to process the data. The first, a multi-input frequency response method, uses the extrapolated rigid body motion at the base of each transducer as a multiple input for estimating frequency response functions between these inputs and the transducer output. Since, this method has been described in detail in previous IMAC papers [ref. 12], the mathematical procedure will not be described in this paper. In should be noted that this method does measure all the cross terms due to both translation and rotation and can be used to calibrate rotational transducers. The second method is computationally simpler, it sets up an eigenvalue solution whose eigenvector is the sensitivities of the transducers. Since the second method has not been documented in the literature, the mathematical derivation of the algorithm is given in Appendix I.

Conclusions-

Spanning the last ten years, the development of multi-channel modal testing continues to receive significant research effort. No longer emphasizing the need to accommodate to temporal parameter estimation methods, research shifts towards spatially managed parameter estimation algorithms. However, the desire to acquire the best, quality, consistent data base remains intact.

Contributing significant advances toward data consistency, recent developments in sensor technology, signal conditioning and signal processing reduce per channel cost to a practical range. The availability of affordable measurement systems creates a competitive forum for sine testing to compete with the traditional broad band methods. The slow sine sweep time no longer hinders a test when researchers instrument hundreds of data points in parallel.

Practical aspects of managing and maintaining large numbers of channels of data occupy a large amount of research time. Some current sensor systems provide advances in the areas of mounting, wiring, identification and signal conditioning, current research probes for more solutions. Recent work in large scale calibration and efficient signal processing exhibit promise for continued convenience and quality in measurements.

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APPENDIX I
Finding the Calibration Factors

For a given calibration experiment, let \( G \) be a matrix of rigid body modes that correspond to the transducer locations and orientations. Note that the rigid modes are functions of geometry only. If the frequency range of interest in the calibration trespasses on the first flexible modes of the calibration fixture, it may be of advantage to adjoining analytical estimates of the first flexible modes to the \( G \) matrix. Furthermore, assume that \( G \) has been orthonormalized with respect to a prior lumped mass matrix \( M \),

\[ G^T \cdot M \cdot G = I \quad (1) \]

This mass matrix may certainly be chosen to be the identity matrix, or any other convenient weighting matrix. The underlying model assumption is that the transducers readings are correct, up to a possibly frequency dependent scale, or calibration factor per channel. This may be expressed in mathematical form by,

\[ X \cdot S + G \cdot k = 0, \quad (2) \]

where \( X \) is a diagonal matrix of measurements at a given frequency, \( S \) is a vector of calibration factors, and \( k \) is a vector of participation factors for the mode matrix \( G \). In other words, the measurements \( X \), scaled by the calibration factors \( S \) may be expressed as a linear combination of the mode shapes in the matrix \( G \). Now, making \( n \) measurements with several excitation conditions, measurement \#1 being denoted by the diagonal matrix \( X_1 \), we observe that equation (2) applies with a constant mode matrix \( G \) and constant calibration vector \( S \), but with a participation vector \( k_1 \) which is a function of the measurement number. Using equation (2), all data for this frequency may be related as:

\[
\begin{bmatrix}
X_1 & G & \ldots & 0 & \ldots & S \\
X_1 & 0 & \ldots & G & \ldots & k_1 \\
X_n & \ldots & \ldots & \ldots & \ldots & k_n
\end{bmatrix}
\begin{bmatrix}
S \\
k_1 \\
k_n
\end{bmatrix}
= 
\begin{bmatrix}
E_1 \\
E_i \\
E_n
\end{bmatrix}
\quad (3)
\]

where \( E \), the right hand side, is the quantity to be minimized to estimate the calibration factors. Since \( E \) is a vector, what we minimize is a norm of \( E \), for example an elliptical norm given by:

\[ \left| \left| E \right| \right|^2_M = \sum_{i=1}^{n} E_i^T M E_i \quad (4) \]

Where \( M \) is the weighting matrix of equation (1). A nontrivial solution is found by minimizing the equation (4) subject to a normalizing condition on \( S \) and the \( k_i \), such as:

\[ K_i K = \sum_{i=1}^{n} k_i^2 k_i = 1 \quad (5) \]

where:

\[ K = \begin{bmatrix}
k_1 \\
k_i \\
k_n
\end{bmatrix} \]

Inserting equation (3) into equation (4) and simplifying, we obtain:

\[ \left| \left| E \right| \right|^2_M = \left[ S^T K_i^T A B I S \right] K \quad (6) \]

where:

\[ A = \sum_{i=1}^{n} X_i^T M X_i \]

and

\[ B = \begin{bmatrix}
X_1 M G & \ldots & X_n M G
\end{bmatrix} \]

Minimizing equation (6) with respect to equation (5) is seen to be a Rayleigh quotient problem whose solution is given by the smallest regular eigenvalue and corresponding eigenvector of:

\[ \begin{bmatrix}
A & B \\
B & I
\end{bmatrix} \begin{bmatrix}
S \\
K
\end{bmatrix} = \begin{bmatrix}
0 & 0 \\
0 & I
\end{bmatrix} \begin{bmatrix}
S \\
K
\end{bmatrix} \Lambda \quad (7) \]

We note that the matrix \( A \) is diagonal, so we can do an inexpensive exact condensation by letting

\[ S = A^{-1} B K \quad (8) \]

which gives us the smaller condensed eigenvalue problem,
(I - B^t A^{-1} B) \mathbf{K} = \mathbf{K}\Lambda \quad (9)

from which the smallest eigensolution is found by inverse iteration. The calibration factors are now computable from equation (8). Normalizing conditions other than equation (5) may be reasonable when more prior knowledge of the measurement is available, in which case a subspace iteration is more suitable for solving the Rayleigh quotient than the simple condensation we have presented here.

APPENDIX II

The Four point discrete fourier transform

The fourier transform is essentially a bridge between a time signal and its frequency content. As with any transform, the objective is to convert complex (difficult) mathematical operations into simple mathematical operations without adding or losing any information. When used judiciously, the DFT saves tremendous amounts of computational computer power while presenting data in useful fashion. By sampling four points along a sine wave, the transform weighting functions conveniently become zeros and ones.

To understand this, examine the following equation for the fourier transform.

\[ X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt \quad (1) \]

where \( X(\omega) \) = frequency signal
\( x(t) \) = time signal

which can also be expressed as a sum of real and imaginary components,

\[ X(\omega) = \int_{-\infty}^{\infty} x(t) \cos(\omega t) dt - j \int_{-\infty}^{\infty} x(t) \sin(\omega t) dt \quad (2) \]

Unfortunately, since real time signals can rarely be described analytically, the fourier series representation proves more useful. Consider a truncated time signal on the period \( 0 \rightarrow T \). Examining the fourier series representation of \( x(t) \),

\[ x(t) = \sum_{n=1}^{4} A_n \cos(n\omega_0 t) + B_n \sin(n\omega_0 t) \quad (3) \]

where
\[ \omega_0 = \frac{2\pi}{T} \]
\[ T = \text{period of signal} \]
\[ A_0 = \text{DC component of signal} \]
\[ A_n \& B_n = \text{fourier coefficients of signal} \]

Now, if a finite number of terms are considered (typically 512 or 1024) and applied to the fourier transform, the resultant form is the discrete fourier transform. However, the restricted conditions pose some important implications. Since a finite number of terms have been chosen, only a finite number of frequencies can be expressed.

The final form of the discrete fourier transform can be written as,

\[ X(\omega) = \sum_{n=1}^{512} x(t_n) \cos(\omega_0 t_n) + j \cdot x(t_n) \sin(\omega_0 t_n) \quad (4) \]

where
\[ x(t_n) = \text{discrete time samples} \]
\[ \omega_0 = \frac{2\pi}{T} \]
\[ T = \text{period of signal} \]

Clearly, it can be seen that over the period \( T \), \( \sin(\omega_0 t) \) and \( \cos(\omega_0 t) \) are simply orthogonal weighting functions applied to the sampled time signal, \( x(t) \). If the time sampling rate is chosen so that there will be only four samples per wave length, the weighting functions reduce to zeros and ones. (See fig. 3) However, since the 4 pt. DFT limits the frequency span to just the second harmonic of the sine wave, it is necessary to apply anti-alias filtering to the data, eliminating the higher harmonics.

Hence, the imaginary part of equation \( (4) \) reduces to,

\[ \text{Im}[X(\omega)] = \sum_{n=1}^{4} x(t_n) \sin(\omega_0 t_n) \quad (5) \]
which further reduces to,
\[ \text{Im}[X(\omega)] = x(t_2) - x(t_4) \]  \hspace{1cm} (6)

The real part of \( X(\omega) \) reduces to,
\[ \text{Re}[X(\omega)] = \sum_{n=1}^{\infty} x(t_n) \cos(\omega t_n) \]  \hspace{1cm} (7)

and further to,
\[ \text{Re}[X(\omega)] = x(t_1) - x(t_3) \]  \hspace{1cm} (8)

Thus, for 512 channels of data, only 2048 pieces of data need to be processed and stored in computer memory. The transformation into the frequency domain requires only 1024 additions. Interestingly, with an extra eight additions per channel, adaptive processing can be implemented utilizing the signal to noise ratio. This would allow the system to actively determine the number of averages needed to produce acceptable results at each frequency.

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