Application of Independent Component Analysis and Blind Source Separation Techniques to Operational Modal Analysis

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Nomenclature

t	Time
$x(t)$	Vector of output observations
A	Mixing system or mixing matrix
$s(t)$	Source signals
n	Number of sources
$\hat{R}_x$	Covariance matrix
Λ	Eigenvalues of covariance matrix
V_s	Eigenvectors associated with principle eigenvalues
V_n	Eigenvectors associated with noise
$\bar{x}$	Pre-whitened vector of output observations
C	Quadricovariance Matrix
p	Number of time lag
Φ_r	Modal vector associated with r^{th} mode
η_r	Modal coordinates associated with r^{th} mode
Ψ	Modal filter matrix
$i.i.d.$	Independent and identically distributed

Abstract

Independent Component Analysis (ICA) / Blind Source Separation (BSS) is an emerging research area in the field of signal processing. The goal of ICA / BSS techniques is to identify statistically independent and non-Gaussian sources from a linear mixture of such sources. It also extracts the unknown mixture matrix in the process. The purpose of this paper is to explore the possibilities of utilizing this concept of ICA for the purpose of Operational Modal Analysis in which the system is identified only on the basis of output responses which are a function of unknown input forces and underlying system characteristics. Four different ICA algorithms are used in this study by applying them on a 15 degrees-of-freedom analytical system. These algorithms differ from each other on the basis of the optimizing techniques implemented and by utilization of second or higher order statistics. The modal parameters estimated using these methods are compared with true system parameters and also with one of the Operational Modal Analysis techniques.

1. Introduction

Experimental Modal Analysis (EMA) has been the most prominent tool for characterizing the dynamics of a structure. Modal parameter estimation is one of the key stages of overall EMA process where modal parameters namely natural frequency, associated damping, mode shapes and scaling factors that characterize the structure

are estimated. In past, several algorithms have been developed to estimate the modal parameters. Details of these algorithms can be found in [1-4]. However, difficulties in characterizing large structures (such as bridges and buildings) and simulating exact operating conditions (such as those encountered by vehicles on road) led to the development of Operational Modal Analysis in which the system parameters are identified on the basis of the output responses only. This approach is different from the traditional EMA in which both the output responses and the input forces are required to be measured. As in case of EMA, there are several OMA parameter estimation techniques, most of which are basically the extension of popular EMA algorithms to OMA domain [5].

The focus of this paper is Independent Component Analysis (ICA) and its applicability to Operational Modal Analysis. Independent component analysis and other related problems such as Blind Source Separation (BSS), Blind Signal Extraction (BSE) and Multichannel Blind Deconvolution (MBD) share the same generalized blind signal processing problem where the aim is to estimate the original source signal and corresponding mixing matrix based only on the knowledge of mixed output signals. There are several good resources that explain the concept of ICA, BSS and other related problems [6-10]. ICA based methods have found application in diverse areas such as biomedical signal analysis (EEG, MMG etc.), speech enhancement, image processing, wireless communication etc. ICA based methods have also found application in structural dynamics related areas such as damage detection and fault diagnosis [11, 12], rotating machinery vibration [13] etc. Recently, Kerschen et al [14, 15] showed that ICA can also be used for parameter estimation purposes. The work in this paper, illustrates the effectiveness of ICA techniques for the purpose of output only OMA applications. Four different ICA algorithms namely, AMUSE [16], Second Order Blind Identification (SOBI) [17], Joint Approximate Diagonalization (JADE) [18] and Fourth Order Blind Identification (FOBI) [19, 20] are applied to an analytical system and their performance is compared with theoretical and OMA based results.

The paper is organized in the following manner. Section 2 describes the general ICA problem and the various ICA algorithms as listed above briefly. Extension of ICA techniques to OMA and related issues are discussed next. In section 3, a case study, an analytical system, is presented and associated results are discussed. Finally, based on the obtained results, conclusions are presented along with the scope of ICA techniques in the field OMA.

2. Independent Component Analysis

Independent Component Analysis (ICA) or Blind Source Separation (BSS) can be seen as an extension to Principal Component Analysis (PCA) which aims at recovering the source signals from the set observed instantaneous linear mixtures without any *a priori* knowledge of the mixing system.

Mathematically, ICA problem can be formulated as

$$x(t) = As(t) \tag{01}$$

where *x(t)* is a column vector of m output observations representing an instantaneous linear mixture of source signals *s(t)* which is a column vector of *n* sources at time instant *t*. A is an *m X n* matrix referred to as "mixing system" or more commonly as "mixing matrix".

Although ICA and BSS techniques claim to identify both the source signals and the mixing matrices, they do so within certain indeterminacies that include arbitrary scaling, permutation and delay of estimated source signals. However, in spite of these limitations the waveform of the original signal is preserved and in many applications knowledge of source waveform is the most relevant information.

The task of estimating both *s* and *A*, the two unknowns in the above mentioned problem requires certain assumptions to be made about the statistical properties of the sources s_i. ICA assumes that the sources s_i are statistically independent and that they have non-Gaussian distribution. A detailed discussion of the subject of ICA is beyond the scope of this paper; interested readers can refer to [6-10,21-22] for general introduction and details of the various aspects of ICA. In addition to these resources readers can also refer to the special issue on ICA and BSS published by *Mechanical Systems and Signal Processing* in 2005 [23].

2.1 ICA / BSS Algorithms

A wide variety of ICA algorithms are available in the literature [6-8]. These algorithms differ from each other on the basis of the choice of objective function and selected optimization scheme. Although the assumption about statistical independence requires the sources to be non-Gaussian in order to utilize the higher-order statistics (HOS) based BSS methods, several second-order statistics (SOS) based techniques are also available. SOS methods exploit weaker conditions for separating the sources assuming that they have a temporal structure with different autocorrelation functions (or power spectra).

In this section, four ICA / BSS methodologies are briefly discussed.

2.1.1 Algorithm for Multiple Unknown Signals Extraction (AMUSE) [6, 16]

SOS based algorithms like AMUSE assume that:
1. The mixing matrix *A* is of full column rank.
2. Sources are spatially uncorrelated with different autocorrelation functions but are temporally correlated (colored) stochastic signals with zero-mean.
3. Sources are stationary signals and / or second order non-stationary signals i.e. their variances are time varying.

The AMUSE algorithm is outlined below [6]:
1. Estimate the covariance (mean removed correlation) matrix of the output observations

$$\hat{R}_x(0) = \frac{1}{N} \sum_{k=1}^{N} x(k) x^T(k) \tag{02}$$

   where $\hat{R}_x(0)$ is the covariance matrix at zero time lag and *N* is the total number of time samples taken.

2. Compute EVD (or SVD) of $\hat{R}_x(0)$

$$\hat{R}_x(0) = U_x \Sigma_x V_x^T = V_x \Lambda_x V_x^T = V_s \Lambda_s V_s^T + V_N \Lambda_N V_N^T \tag{03}$$

   where V_s is *m X n* matrix of eigenvectors associated with *n* principal eigenvalues of Λ_s = diag{λ_1, λ_2, ..., λ_n} in descending order. V_n is *m X (m-n)* matrix containing the (m-n) noise eigenvectors associated with noise eigenvalues Λ_n = diag{λ_{n+1}, λ_{n+2}, ..., λ_m}. The number of sources *n* are thus estimated based on the *n* most significant eigenvalues (or singular values in case of SVD).

3. Perform pre-whitening transformation

$$\bar{x}(k) = \Lambda_s^{-\frac{1}{2}} V_s^T x(k) = Qx(k) \tag{04}$$

4. Estimate the covariance matrix of the vector $\bar{x}(k)$ for specific time lag other than p=0 (say p=1). Perform SVD on the estimated covariance matrix.

$$\hat{R}_{\bar{x}}(p) = \frac{1}{N} \sum_{k=1}^{N} \bar{x}(k) \ \bar{x}^T(k-p) = U_{\bar{x}} \Sigma_{\bar{x}} V_{\bar{x}}^T \tag{05}$$

5. The mixing matrix and source signals can be estimated as

$$\hat{A} = Q^+ U_{\bar{x}} = V_s \Lambda_s^{\frac{1}{2}} U_{\bar{x}} \tag{06}$$

$$y(k) = \hat{s}(k) = U_{\bar{x}}^T \bar{x}(k) \tag{07}$$

AMUSE performs well for colored sources with different power spectra shapes which means that the eigenvalues of the time-delayed covariance matrix are distinct. The accuracy of AMUSE however deteriorates in presence of additive noise.

2.1.2 Fourth Order Blind Identification Algorithm (FOBI) [6, 7, 19-20]

FOBI is an extension of the AMUSE algorithm which uses contracted quadricovariance matrices instead of covariance matrices. This technique is aimed at separating independent non-Gaussian source signals. The quadricovariance matrices are defined as

$$C_{\bar{x}}(\mathbf{E}) = C_{\bar{x}} \{ \bar{x}^T(k) \mathbf{E} \bar{x}(k) \bar{x}(k) \bar{x}^T(k) \}$$
$$= E\{ \bar{x}^T(k) \mathbf{E} \bar{x}(k) \bar{x}(k) \bar{x}^T(k) \} - R_{\bar{x}}(0) \mathbf{E} R_{\bar{x}}(0) - tr(\mathbf{E} R_{\bar{x}}(0)) R_{\bar{x}}(0) - R_{\bar{x}}(0) \mathbf{E}^T R_{\bar{x}}(0) \tag{08}$$

where $R_{\bar{x}}(0) = E\{ \bar{x}(k) \bar{x}^T(k) \}$ and **E** is an *m X n* freely chosen matrix called eigenmatrix (typically **E** = $\mathbf{I}_n$ or **E** = $\mathbf{e}_q \mathbf{e}_q^T$, where $\mathbf{e}_q$ are vectors of some unitary matrix).

The eigenvalue decomposition of the quadricovariance matrix is of the following form

$$C_{\bar{x}}(\mathbf{E}) = U \Lambda_{\mathbf{E}} U^T \tag{09}$$

with $\Lambda_E = diag(\lambda_1 u_1^T \mathbf{E} u_1,.........,\lambda_n u_n^T \mathbf{E} u_n)$, $\lambda_i = \kappa_4(s_i) = E\{s_i^4\} - 3E^2\{s_i^2\}$ is the kurtosis of the zero-mean i^{th} source and u_i is the i-th column of the orthogonal eigenvector matrix U.

The main advantage of the FOBI algorithm is that it is insensitive to arbitrary Gaussian noise and that it allows the mixing system to be identified when sources are i.i.d. and mutually independent. However, it should be noted that quadricovariance matrices require many more time samples for correct estimates in comparison to covariance matrices. FOBI also has a restriction that it only works for sources having different kurtosis; thus it will not give good results in cases where sources have identical distributions.

2.1.3 Second Order Blind Identification (SOBI) [6, 17]

SOBI algorithm utilizes the joint diagonalization procedure [6, 18, 24] unlike AMUSE and FOBI which use EVD / SVD techniques. SOBI works well for simple colored sources with distinct power spectra (or distinct autocorrelation functions). Like AMUSE, it operates on time delayed covariance matrices.

SOBI utilizes the pre-whitening transformation similar to that described in case of AMUSE. This is followed by estimation of set of covariance matrices for a preselected set of time lags (p_1, p_2, ..., p_L)

$$\hat{R}_{\bar{x}}(p_i) = \frac{1}{N}\sum_{k=1}^{N}\bar{x}(k)\bar{x}^T(k - p_i) = Q\hat{R}_{\bar{x}}(p_i)Q^T \tag{10}$$

Joint approximate diagonalization (JAD) is performed on the above matrices; $R_{\bar{x}}(p_i) = UD_iU^T$, to estimate the orthogonal matrix U that diagonalizes a set of covariance matrices. Several efficient algorithms are available for this purpose including Jacobi techniques, Alternating Least Squares, Parallel Factor Analysis etc. [18, 24]. Finally the sources and signals can be estimated using the same equations as explained earlier with AMUSE.

It should be noted that $D_i(p_i)$ is a diagonal matrix that has distinct diagonal entries. However, it is difficult to determine a priori a single time lag p at which the above criterion is satisfied. Joint diagonalizaton procedure avoids this difficulty by providing an optimum solution considering a number of time lags.

2.1.4 Joint Approximate Diagonalization of Eigenmatrices (JADE) [6, 25]

JADE can be considered as an extension of SOBI and FOBI algorithms. Like FOBI, JADE works on the contacted quadricovariance matrices but instead of employing EVD / SVD it jointly diagonalizes a set of such matrices just like SOBI. The aim of JADE is to estimate an orthogonal matrix U which diagonalizes a set of quadricovariance matrices. JADE is a mathematically intensive algorithm and the complete explanation is beyond the scope of this paper. Interested readers can refer the above mentioned references for more details.

3. ICA and BSS in Vibrations

Due to the tremendous potential of ICA / BSS techniques, it is not a surprise that research community in the vibration associated field have also started looking at utilizing them for variety of purposes. In [13], second order ICA techniques were utilized for rotating machinery vibration analysis. ICA of vibration signals was also used for fault diagnosis of an induction motor [12]. ICA was used along with Artificial Neural Network (ANN) for data reduction purposes while detecting structural damage [11]. However in spite of their tremendous potential, use of ICA and BSS techniques in vibration and related areas has been slow in comparison to some of the other areas. Antoni [26] has discussed the issues associated with application of ICA / BSS techniques for vibration signals in detail. One of the major issues with the application of ICA / BSS techniques to vibrations, particularly structural identification related applications, is the fact that vibrating systems are dynamic or convolutive in nature as opposed to static (instantaneous linear) mixtures for which the ICA / BSS theory is originally designed. One obvious way to tackle the convolutive mixtures is to deal with them in the frequency domain as convolution in time domain is equivalent to multiplication in frequency domain. However, there are two other problems which are inherent to ICA / BSS techniques; scaling of sources and the order in which they are identified, often referred as 'permutation problem'. This is the same problem encountered in estimating partial coherence and/or conditioned partial coherence over 20 years ago with respect to partially dependent sources in acoustics (general MIMO problem) and in multiple input excitation problems in structural dynamics (MIMO-FRF estimation). In the case of frequency domain ICA / BSS, these two problems become much more severe as they now become a function of each frequency bin. Frequency domain ICA / BSS is a topic of ongoing research and lots of efforts are being made to understand it [27-30]. Most of these algorithms were based on the fact that convolved mixing in time domain corresponds to instantaneous mixing in frequency domain. The work done in this aspect deals with

handling of the scaling and permutation problems. However so far no significant success has been achieved in this aspect and research is still going on.

3.1 ICA / BSS techniques for Operational Modal Analysis

Operational Modal Analysis is an emerging technique in the field of modal analysis where dynamic characteristics of a system are identified based only on the output responses. OMA has found applications in the areas of civil structures, automobile, rotating machinery etc. Since, by definition, ICA / BSS techniques works only on system outputs to identify either the sources or the system (mixing matrix) without any a priori (or very little) information about them, it is logical to believe that these techniques can also be used for OMA purposes. Recently, it was shown how ICA / BSS techniques can be utilized for the purpose of modal analysis [14-15, 31]. The basic fundamentals behind application of ICA / BSS techniques to modal analysis goes back to the concept of expansion theorem [32] and modal filters [33-35]. According to the expansion theorem the response of a distributed parameter structure can be expressed as

$$x(t) = \sum_{r=1}^{\infty} \phi_r \eta_r(t) \tag{11}$$

where Φ_r are the modal vectors weighted by the modal coordinates η_r. For real systems, however, the response of the system can be represented as a finite sum of modal vectors weighted by the modal coordinates. To obtain a particular modal coordinate η_i from response vector x, a modal filter vector ψ_i is required such that

$$\psi_i^T \phi_i = 0, \qquad for \quad i \neq j \tag{12}$$

and

$$\psi_i^T \phi_i \neq 0, \qquad for \quad i = j \tag{13}$$

so that

$$\psi_i^T x(t) = \psi_i^T \sum_{r=1}^{N} \phi_r \eta_r(t) \tag{14}$$

or

$$= \psi_i^T \phi_i \eta_i \tag{15}$$

Thus modal filter performs a coordinate transformation from physical to modal coordinates. Multiplying the system response x with modal filter matrix Ψ^T results in uncoupling of the system response into single degree of freedom (SDOF) modal coordinate responses (η).

In order for ψ_i to exist, the associated modal vector Φ_i must be linearly independent with respect to all other modal vectors [33]. This is also the reason why ICA / BSS based techniques can be utilized for the purpose of decomposing the output system response into a product of modal vectors and corresponding modal coordinate responses. Also, a modal filter vector is unique if and only if the number of sensors used for the modal filter implementation is equal to the number of linearly independent modal vectors contributing to the system response.

In the past, SDOF modal coordinate responses have been obtained by utilizing modal filters calculated using FRF based data. The ICA / BSS based techniques differ from this approach in the sense that they directly work on output system response to obtain the modal coordinate responses (η). This approach is similar to that in [14, 15] where modal coordinates are treated as virtual sources.

In the following section, four different ICA algorithms as described in Section 2.1 are applied to a 15 degrees of freedom system.

4. Analytical 15 Degree of Freedom System

Figure 1 shows a 15 degree of freedom system which is excited at all 15 degrees of freedom by means of a white random uncorrelated set of inputs. The chosen system has some closely spaced modes (around 53 Hz), some modes that are lightly damped, other modes that are moderately damped and also some local modes that are well separated from each other. This makes it a good system to investigate the various ICA / BSS algorithms described in Section 2.

Figure 2-5 show the plot of the auto-power spectrums of the modal coordinate responses (η) obtained using various algorithms. The Second Order Statistics (SOS) based algorithms, AMUSE and SOBI, uncouple the

system response into SDOF modal coordinate responses. However, both Higher Order Statistics (HOS) based methods, JADE and FOBI, fail to successfully separate the response into corresponding modal coordinate response. Possible reasons for the inferior behavior of the HOS based methods can be that quadricovariance matrices are not correctly estimated and also the fact that SOS based methods better exploit the temporal coherence (uniqueness of autopower spectra) of various modal coordinate responses.

Modal parameters obtained using the SDOF response based on AMUSE and SOBI are listed in Table 1 and are compared with true analytical modes of the system and also with the results obtained using OMA-EMIF algorithm [36]. Though the frequency estimates using AMUSE and SOBI is close to the true modes in comparison to the OMA-EMIF algorithm, damping is overestimated for all the modes. However, overall results are satisfactory. Further, Figures 6-9 show, the modal assurance criterion (MAC) plots for comparing the modal vectors obtained using various methods. The modal vectors obtained using AMUSE and SOBI are in good agreement with each other. However the MAC values are not that high when modal vectors obtained using AMUSE are compared with true modes or OMA-EMIF results. The high MAC values for true and OMA-EMIF modal vectors indicate that the OMA-EMIF method is able to extract the modal vectors of the system better in comparison to the ICA techniques. Note that using AMUSE or SOBI, the modes are obtained in a random order. Also, the repeated modes around 53.3 Hz have interchanged when estimated using OMA-EMIF algorithm.
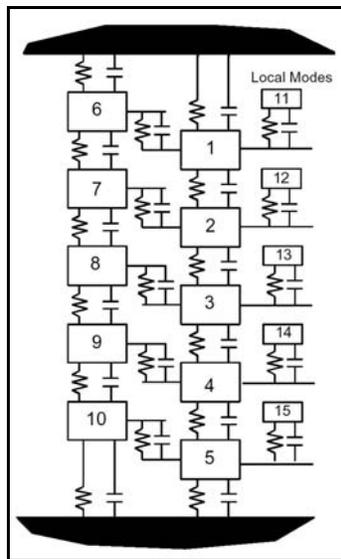

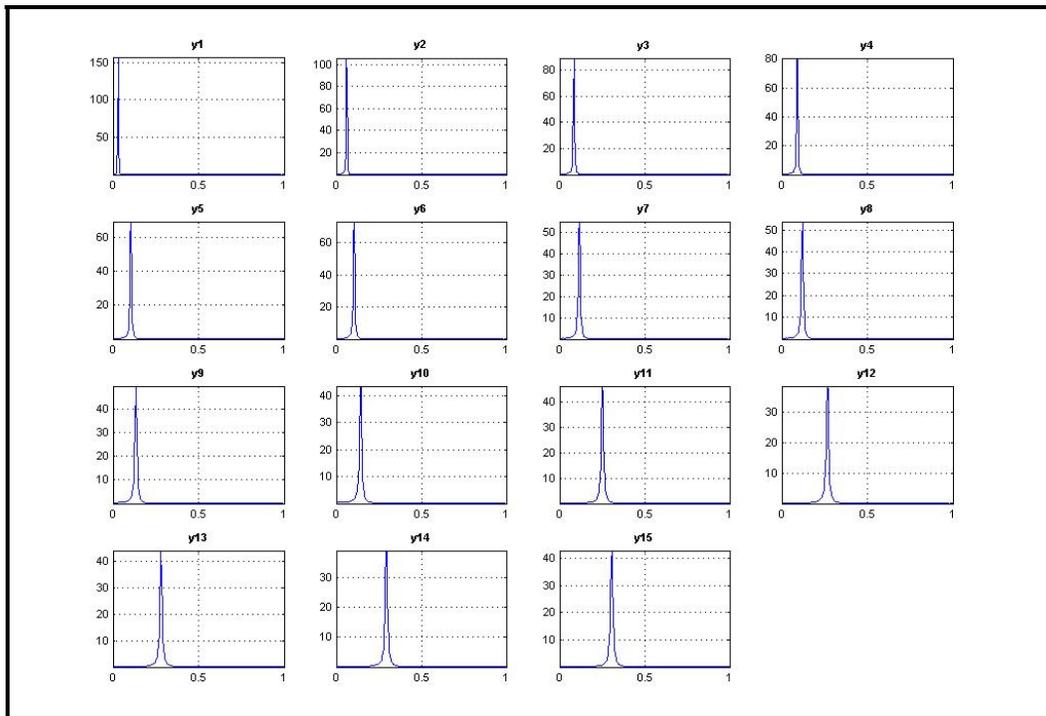
Figure 1: Analytical 15 Degree of Freedom System

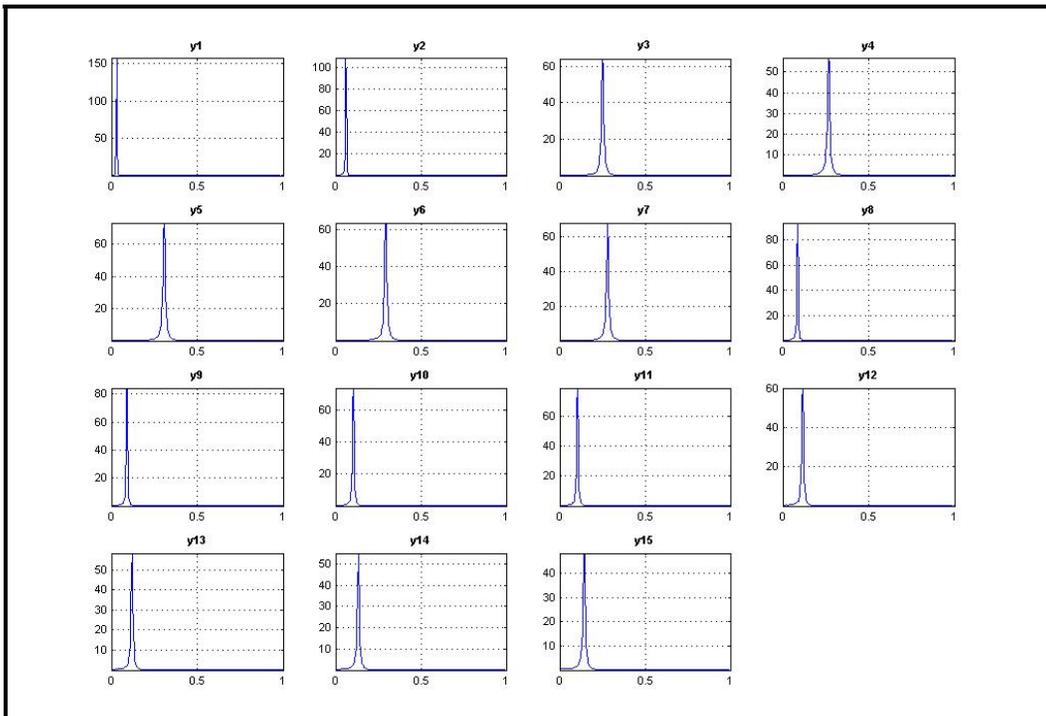**Figure 2: Power Spectrum of Modal Coordinate Responses obtained using AMUSE**

Figure 3: Power Spectrum of Modal Coordinate Responses obtained using SOBI
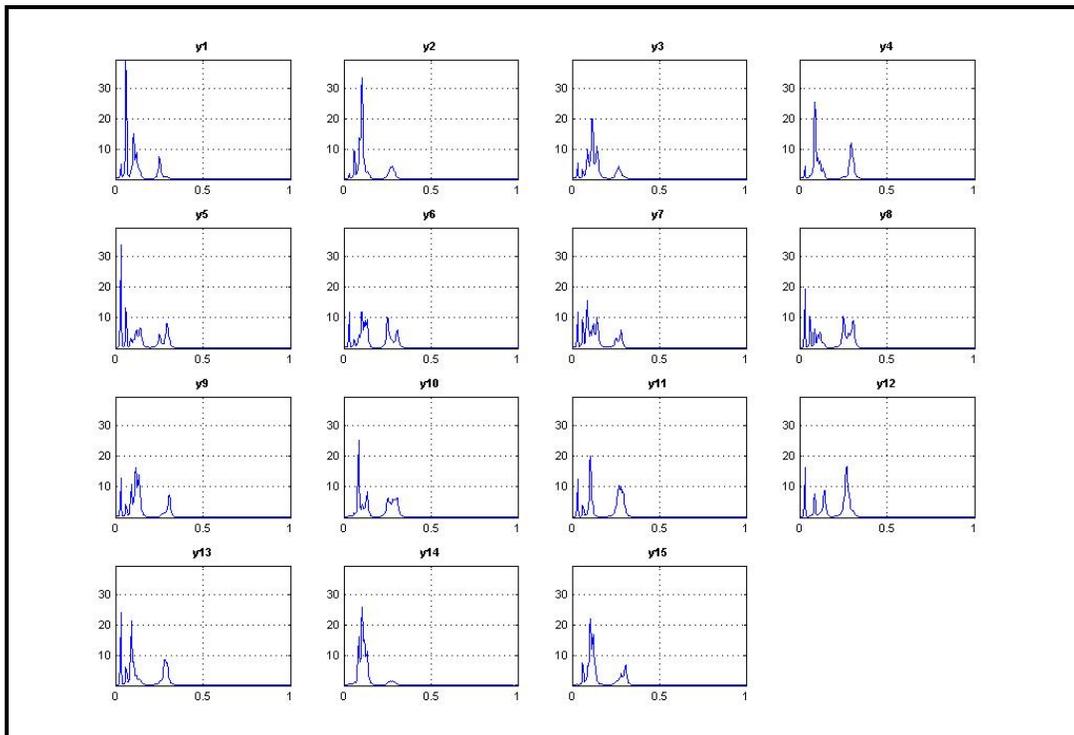
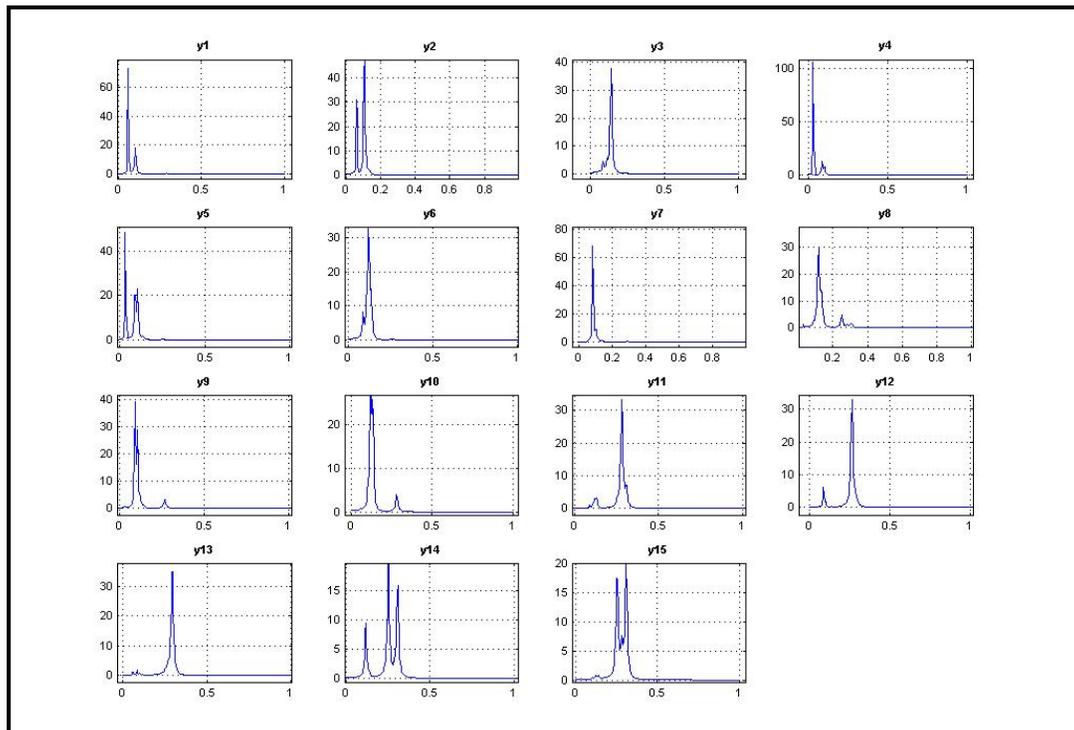**Figure 4: Power Spectrum of Modal Coordinate Responses obtained using FOBI**

Figure 5: Power Spectrum of Modal Coordinate Responses obtained using JADE

Table 1: Comparison of Modal Parameters

| True Modes		OMA-EMIF		ICA - AMUSE		ICA - SOBI	
Damp	Freq	Damp	Freq	Damp	Freq	Damp	Freq
1.0042	15.985	3.3289	15.9857	2.1133	15.9774	2.2314	15.9806
1.9372	30.858	2.8564	30.8619	2.4042	30.8709	2.4123	30.8725
2.7347	43.6	3.2718	43.6932	3.1196	43.6511	3.1441	43.6543
2.9122	46.444	3.6481	46.5883	3.4458	46.4035	3.4787	46.4059
3.3375	53.317	4.0763	53.7487	3.7103	53.3869	3.8181	53.4067
3.3454	53.391	4.7860	53.7683	3.5785	53.4237	3.6199	53.4271
3.7145	59.413	3.6833	59.1164	4.1244	59.4877	4.2735	59.5196
3.858	61.624	3.6157	61.3713	4.3618	61.6476	4.4495	61.6577
4.2978	68.811	4.1523	68.2375	4.5887	68.9075	4.7218	68.9299
4.5925	73.63	4.4249	73.1519	4.8149	73.7923	4.9619	73.8233
2.6093	128.84	2.6146	128.5604	2.710	128.9026	2.7803	128.9105
2.4548	136.55	2.6030	136.4652	2.5743	136.6159	2.6264	136.6253
2.3288	143.86	2.4513	143.8957	2.4323	143.9370	2.4734	143.9487
2.221	150.83	2.3207	150.8782	2.3162	150.9051	2.3589	150.9175
2.122	157.47	2.0669	157.7820	2.2087	157.54	2.2391	157.5533
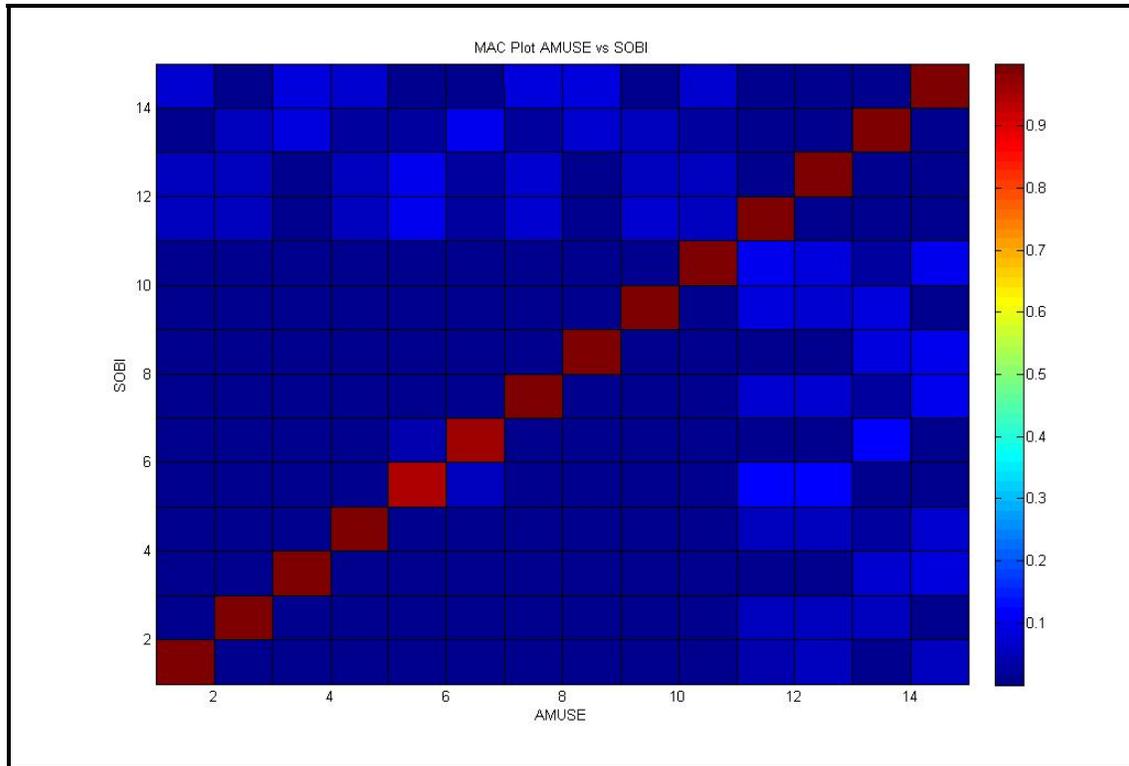
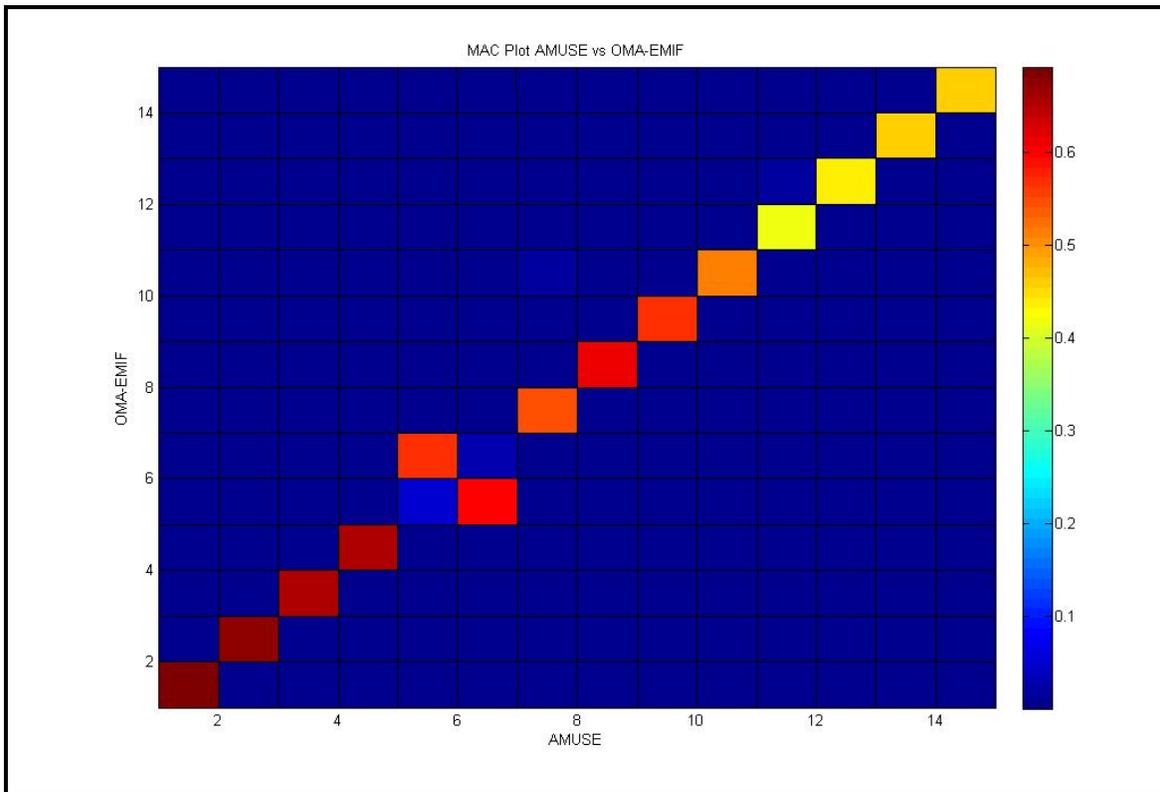**Figure 6: MAC Comparison Plot - AMUSE / SOBI**

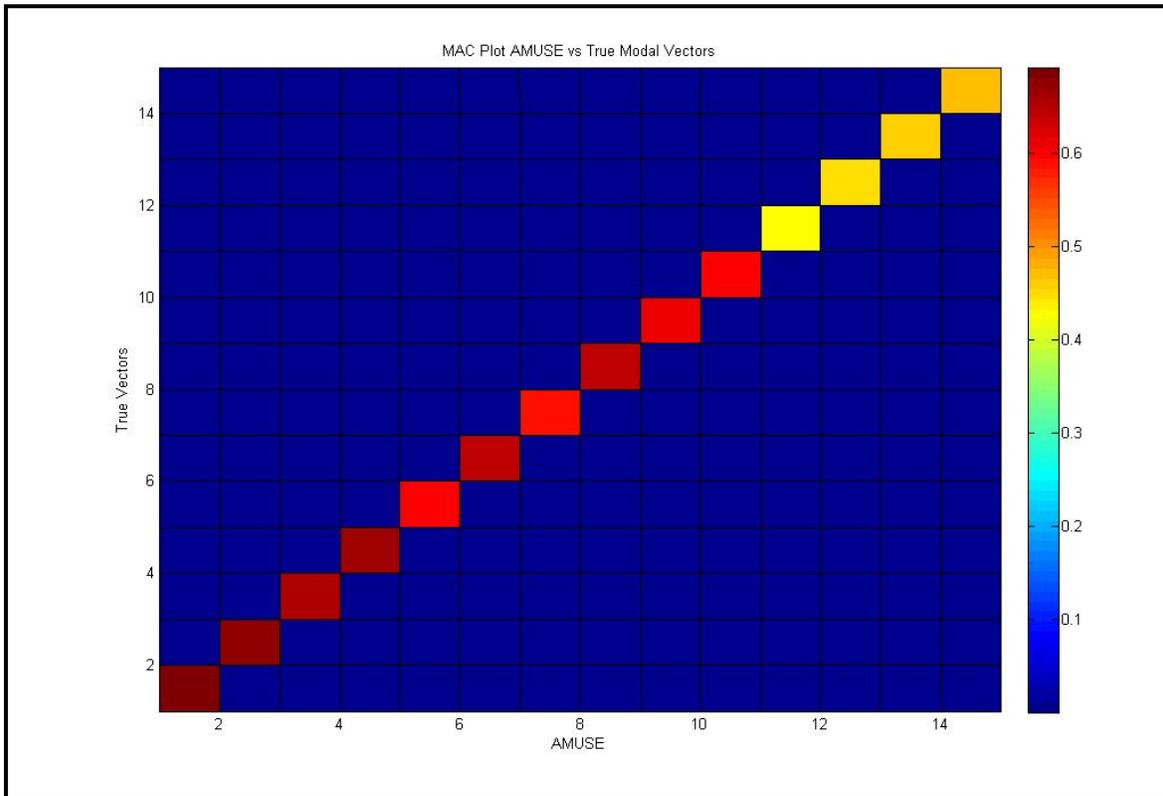**Figure 7: MAC Comparison Plot - AMUSE / OMA-EMIF**

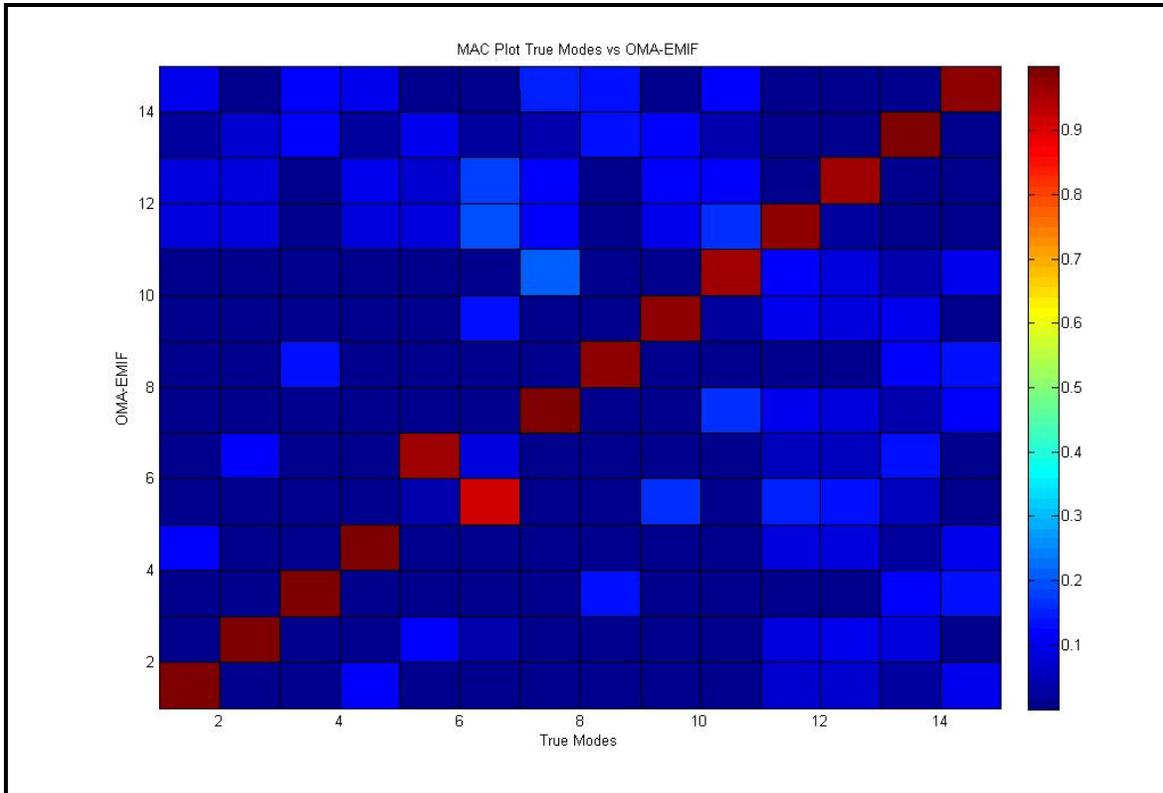**Figure 8: MAC Comparison Plot - AMUSE / True Modes**

Figure 9: MAC Comparison Plot - True Modes / OMA-EMIF

6. Conclusions

The paper discusses four popular ICA / BSS techniques along with the general concept of independent component analysis (ICA) and blind source separation (BSS). It is shown how these techniques can be utilized for output-only modal parameter estimation purposes by relating them to the concepts of modal filtering and modal expansion theorem. The studies conducted on an analytical system reveal that second order statistics based ICA / BSS algorithms give better results in comparison to the higher order statistics based algorithms. Though ICA / BSS based results are not as good as general OMA algorithms based results, it is still an interesting area to explore considering the simplicity of the method and its ability to extract all modal parameters (modal frequencies and mode shapes) in one step. These algorithms are comparatively less time consuming and does not require use of such tools as consistency diagrams. One of the issues with these algorithms is the fact that one should have at least as many sensors to measure the system output as the number of system modes one is interested in.

The future research in this area needs to concentrate on better understanding of the various ICA algorithms for better application to modal analysis. Application to more experimental and real-life structures will also aid in throwing more light on advantages and limitations of these methods. Frequency domain based ICA / BSS is yet another area whose development might result in better implementation of these techniques to OMA.

Acknowledgements

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