ABSTRACT

The use of controlled excitation is fundamental to the acquisition of input-output data for the purpose of estimating modal parameters. The characteristics of the excitation greatly influence the quality of the resulting measurements. This paper first presents an overview of the basic excitation methods, such as, random, periodic random, pseudo-random, and burst random. The paper then reviews the use of cyclic averaging with random excitation methods. Finally, the paper develops a new excitation method that is the result of combining cyclic averaging with burst random excitation. This combination of methods results in an improved signal-to-noise ratio (SNR) for lightly damped structures where leakage is a significant problem. An experimental example is given for a typical lightly damped, structural system.

Nomenclature

\[ N_{avg} = \text{Number of averages}. \]
\[ N_c = \text{Number of cyclic averages}. \]
\[ N_s = \text{Number of spectral averages}. \]
\[ N_i = \text{Number of inputs}. \]
\[ N_o = \text{Number of outputs}. \]
\[ F_{\text{max}} = \text{Maximum frequency (Hz.)}. \]
\[ \Delta f = \text{Frequency resolution (Hz.)}. \]
\[ T = \text{Observation period (Sec.)}. \]

1. Introduction

Single and multiple input estimation of frequency response functions via shaker excitation has become the mainstay of most mechanical structure measurements, particularly in the automotive and aircraft industries. While there are appropriate occasions for the use of deterministic excitation signals (sinusoids), the majority of these measurements are made using broadband (random) excitation signals. These signals work well for moderate to heavily damped mechanical structures which exhibit linear characteristics. When the mechanical structures are very lightly damped, care must be taken to minimize the leakage error so that accurate frequency response function (FRF) data can be estimated in the vicinity of the modal frequencies of the system.

Historically, a number of random excitation signals have been utilized, together with appropriate digital signal processing techniques \[^{[1-5]}\], to obtain accurate FRF data. The most common random signal that is used in this situation is the burst random signal, where the burst length of the random signal is adjusted to a portion of the total observation time (T). The length of the burst random signal is chosen based upon the measured signals being completely observed transients in the observation time (T). This is dependent upon the amount of system damping and the damping characteristic provided by the shaker/amplifier operating in voltage feedback mode. This works well in most cases but, in cases involving very light damping, the burst length becomes so short (10-20% of observation time, T) that the signal-to-noise ratio (SNR) of the measurement starts to suffer. This situation can be adequately addressed by the method presented in the following sections.

2. Background

The background of this method involves a combination of the traditional burst random excitation method with the cyclic averaging technique. These concepts are reviewed in the following sections.

2.1 Random Excitation Methods

Inputs which can be used to excite a system in order to determine frequency response functions belong to one of two classifications, random or deterministic \[^{[6-8]}\]. Random signals are widely utilized for general single-input and multiple-input shaker testing when evaluating structures that are essentially linear. Signals of this form can only be defined by their statistical properties over some time period. Any subset of the total time period is unique and no explicit mathematical relationship can be formulated to describe the signal. Random signals can be further classified as stationary or non-stationary. Stationary random signals are a special case where the statistical properties of the random signals do not vary with respect to translations with time. Finally, stationary random signals can be classified as ergodic or non-ergodic. A stationary random signal is ergodic when a time average on any particular subset of the signal is the same for any arbitrary subset of the random signal. All random signals which are commonly used as input signals fall into the category of ergodic, stationary random signals. Deterministic signals can be characterized directly by
The choice of input to be used to excite a system in order to determine frequency response functions depends upon the characteristics of the system, upon the characteristics of the parameter estimation, and upon the expected utilization of the data. The characterization of the system is primarily concerned with the linearity of the system. As long as the system is linear, all input forms should give the same expected value. Naturally, though, all real systems have some degree of nonlinearity. Deterministic input signals result in frequency response functions that are dependent upon the signal level and type. A set of frequency response functions for different signal levels can be used to document the nonlinear characteristics of the system. Random input signals, in the presence of nonlinearities, result in a frequency response function that represents the best linear representation of the nonlinear characteristics for a given level of random signal input. For small nonlinearities, use of a random input will not differ greatly from the use of a deterministic input.

The characterization of the parameter estimation is primarily concerned with the type of mathematical model being used to represent the frequency response function. Generally, the model is a linear summation based upon the modal parameters of the system. Unless the mathematical representation of all nonlinearities is known, the parameter estimation process cannot properly weight the frequency response function data to include nonlinear effects. For this reason, random input signals are prevalently used to obtain the best linear estimate of the frequency response function when a parameter estimation process using a linear model is to be utilized.

The expected utilization of the data is concerned with the degree of detailed information required by any post-processing task. For experimental modal analysis, this can range from implicit modal vectors needed for trouble-shooting to explicit modal vectors used in an orthogonality check. As more detail is required, input signals, both random and deterministic, will need to match the system characteristics and parameter estimation characteristics more closely. In all possible uses of frequency response function data, the conflicting requirements of the need for accuracy, equipment availability, testing time, and testing cost will normally reduce the possible choices of input signal.

With respect to the reduction of the variance and bias errors of the frequency response function, random or deterministic signals can be utilized most effectively if the signals are periodic with respect to the sample period or totally observable with respect to the sample period. If either of these criteria are satisfied, regardless of signal type, the predominant bias error, leakage, will be minimized.

If these criteria are not satisfied, the leakage error may become significant. In either case, the variance error will be a function of the signal-to-noise ratio and the amount of averaging.

Many signals are appropriate for use in experimental modal analysis. Some of the most commonly used random signals, used with single and multiple input shaker testing, are described in the following sections.

**Pure Random** - The pure random signal is an ergodic, stationary random signal which has a Gaussian probability distribution. In general, the frequency content of the signal contains all frequencies (not just integer multiples of the FFT frequency increment) but may be filtered to include only information in a frequency band of interest. The measured input spectrum of the pure random signal will be altered by any impedance mismatch between the system and the exciter.

**PseudoRandom** - The pseudorandom signal is an ergodic, stationary random signal consisting only of integer multiples of the FFT frequency increment. The frequency spectrum of this signal has a constant amplitude with random phase. If sufficient time is allowed in the measurement procedure for any transient response to the initiation of the signal to decay, the resultant input and response histories are periodic with respect to the sample period. The number of averages used in the measurement procedure is only a function of the reduction of the variance error. In a noise free environment, only one average may be necessary.

**Periodic Random** - The periodic random signal is an ergodic, stationary random signal consisting only of integer multiples of the FFT frequency increment. The frequency spectrum of this signal has random amplitude and random phase distribution. Since a single history will not contain information at all frequencies, a number of histories must be involved in the measurement process. For each average, an input history is created with random amplitude and random phase. The system is excited with this input in a repetitive cycle until the transient response to the change in excitation signal decays. The input and response histories should then be periodic with respect to the observation time (T) and are recorded as one average in the total process. With each new average, a new history, uncorrelated with previous input signals, is generated so that the resulting measurement will be completely randomized.

**Burst Random (Random Transient)** - The burst random signal is neither a completely transient deterministic signal nor a completely ergodic, stationary random signal but contains properties of both signal types. The frequency spectrum of this signal has random amplitude and random phase distribution and contains energy throughout the frequency spectrum. The difference between this signal and the random signal is that the random transient history is truncated to zero after some percentage of the observation time (T). Normally, an acceptable percentage is fifty to eighty percent. The measurement procedure duplicates the random procedure but without the need to utilize a window to reduce the leakage problem. The point that the input history is truncated is chosen so that the response history decays to zero within the observation time (T). For light to moderately damped systems, the response history will decay to zero very quickly due to the damping provided by the
exciter/amplifier system trying to maintain the input at zero (voltage feedback). This damping, provided by the exciter/amplifier system, is often overlooked in the analysis of the characteristics of this signal type. Since this measured input, although not part of the generated signal, includes the variation of the input during the decay of the response history, the input and response histories are totally observable within the sample period and the system damping that will be computed from the measured FRF data is unaffected.

2.2 Cyclic Signal Averaging

The cyclic classification of signal averaging involves the added constraint that the digitization is coherent between sample functions [9-10]. This means that the exact time between each sample function is used to enhance the signal averaging process. Rather than trying to keep track of elapsed time between sample functions, the normal procedure is to allow no time to elapse between successive sample functions. This process can be described as a comb digital filter in the frequency domain with the teeth of the comb at frequency increments dependent upon the periodic nature of the sampling with respect to the event measured. The result is an attenuation of the spectrum between the teeth not possible with other forms of averaging.

This form of signal averaging is very useful for filtering periodic components from a noisy signal since the teeth of the filter are positioned at harmonics of the frequency of the sampling reference signal. This is of particular importance in applications where it is desirable to extract signals connected with various rotating members. This same form of signal averaging is particularly useful for reducing leakage during frequency response measurements and also has been used for evoked response measurements in biomedical studies.

A very common application of cyclic signal averaging is in the area of analysis of rotating structures. In such an application, the peaks of the comb filter are positioned to match the fundamental and harmonic frequencies of a particular rotating shaft or component. This is particularly powerful, since in one measurement it is possible to enhance all of the possible frequencies generated by the rotating member from a given data signal. With a zoom Fourier transform type of approach, one shaft frequency at a time can be examined depending upon the zoom signal. With a zoom Fourier transform type of approach, one shaft frequencies generated by the rotating member from a given data measurement it is possible to enhance all of the possible component. This is particularly powerful, since in one and harmonic frequencies of a particular rotating shaft or peaks of the comb filter are positioned to match the fundamental area of analysis of rotating structures. In such an application, the

The Fourier transform of a history is given by:

$$X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt \quad (1)$$

Using the time shift theorem of the Fourier transform, the Fourier transform of the same history that has been shifted in time by an amount $$t_0$$ is:

$$X(\omega) e^{-j\omega t_0} = \int_{-\infty}^{\infty} x(t + t_0) e^{-j\omega t} dt \quad (2)$$

For the case of a discrete Fourier transform, each frequency in the spectra is assumed to be an integer multiple of the fundamental frequency $$\Delta f = \frac{1}{T}$$. Making this substitution in Equation (2) ($$\omega = n \frac{2\pi}{T}$$ with $$n$$ as an integer) yields:

$$X(\omega) e^{-j\frac{2\pi}{T} t_0} = \int_{-\infty}^{\infty} x(t + t_0) e^{-j\omega t} dt \quad (3)$$

Note that in Equation (3), the correction for the cases where $$t_0 = N T$$ with $$N$$ an integer will be a unit magnitude with zero phase. Therefore, if each history that is cyclic averaged occurs at a time shift, with respect to the initial average, that is an integer multiple of the observation period $$T$$, then the correction due to the time shift does not effect the frequency domain characteristics of the averaged result. All further discussion will assume that the time shift $$t_0$$ will be an integer multiple of the basic observation period $$T$$.

The signal averaging algorithm for histories averaged with a boxcar or uniform window is:

$$\bar{x}(t) = \frac{1}{N_c} \sum_{i=0}^{N_c-1} x_i(t) \quad (4)$$

where:

2.2.1 Theory of Cyclic Averaging

In the application of cyclic averaging to frequency response function estimates, the corresponding fundamental and harmonic frequencies that are enhanced are the frequencies that occur at the integer multiples of $$\Delta f$$. In this case, the spectra between each $$\Delta f$$ is reduced with an associated reduction of the bias error called leakage.

The first observation to be noted is the relationship between the Fourier transform of a history and the Fourier transform of a time shifted history. In the averaging case, each history will be of some finite time length $$T$$ which is the observation period of the data. Note that this time period of observation $$T$$ determines the fundamental frequency resolution $$\Delta f$$ of the spectra via the Rayleigh Criteria ($$\Delta f = \frac{1}{T}$$).

The result is an attenuation of the spectrum between the teeth not possible with other forms of averaging.
• $x_i(t)$ = Time history, average $i$

• $N_c$ = Number of cyclic averages

For the case where $x(t)$ is continuous over the time period $N_cT$, the complex Fourier coefficients of the cyclic averaged time history become:

$$ C_k = \frac{1}{T} \int_0^T x_i(t) e^{-j\omega t} dt \quad (5) $$

$$ C_k = \frac{1}{T} \int_0^T \frac{1}{N_c} \sum_{i=0}^{N_c-1} x_i(t) e^{-j\omega t} dt \quad (6) $$

Finally:

$$ C_k = \frac{1}{N_c T} \int_0^{N_cT} x(t) e^{-j\omega t} dt \quad (7) $$

Since $x(t)$ is a continuous function, the sum of the integrals can be replaced with an integral evaluated from 0 to $N_c T$ over the original function $x(t)$. Therefore:

$$ C_k = \frac{1}{N_c T} \int_0^{N_cT} x(t) e^{-j\omega t} dt \quad (8) $$

The above equation indicates that the Fourier coefficients of the cyclic averaged history (which are spaced at $\Delta f = \frac{1}{T}$) are the same Fourier coefficients from the original history (which are spaced at $\Delta f = \frac{1}{N_c T}$).

The results of cyclic averaging of a general random signal with the application of a uniform window are shown in Figures 5 and 6. Likewise, the results of cyclic averaging of a general random signal with the application of a Hann window are shown in Figures 7 and 8.

3. New Method

The new method developed in this paper is simply a combination of burst random excitation and cyclic averaging. Burst random excitation is useful whenever a leakage problem exists in making a frequency response function measurement on a lightly damped mechanical system. The limitation of burst random excitation is that, if the burst length is shortened too far (10-20 percent of the observation time block, $T$) in order to minimize the leakage problem, the signal-to-noise ratio (SNR) of the excitation signal deteriorates. Cyclic averaging allows the burst random signal to exist over a large portion of the contiguous observation time and yet permits the burst length of random signal to be adjusted to any time length needed. Note that this combination of techniques allows leakage to be reduced in two ways while maintaining a reasonable SNR: 1) the burst length can be adjusted as needed and
The following figures. Figure 9 shows four contiguous observation times for a burst random signal measured at the load cell and a typical response transducer. Figure 10 is the cyclic average of these four blocks. This becomes the first average for any spectral averages that will be accumulated. Note that in this case, even though the burst length is chosen so that the excitation signal will be zero after 20 percent of the fourth block of the four contiguous averages, the response signal, shown in Figure 9, still has not decayed to zero by the end of the fourth block.

4. Structural Example

The following example represents a single measurement on an H-frame test structure in a test lab environment. The test results are representative of all data taken on the H-frame structure. This H-frame test structure is very lightly damped and has been the subject of many previous studies.

Nine representative cases were measured on this structure. The configuration of the test involved two shaker locations (inputs) and eight response accelerometers (outputs). The frequency response function measurements presented in the following discussion were calculated utilizing both the $H_1$ and $H_v$ algorithm approaches. The $H_1$ data is shown in the plots. The digital signal processing characteristics of each case are shown in Table 1. In all cases, the total data acquired and hence the total test time are the same. In this case 96 data ensembles are used simply changing the relative number of cyclic and spectral averages.

Case 1 is considered a baseline case since this a very popular method for making a FRF measurement. However, it is clear that in this measurement situation, there is a significant drop in the multiple coherence function at frequencies consistent with the peaks in the FRF measurement. This characteristic drop in multiple (or ordinary) coherence is often an indication of a leakage problem. This can be confirmed if a leakage reduction method reduces or eliminates the problem when the measurement is repeated. In all subsequent cases, the test configuration was not altered in any way - data was acquired simply using different burst random, cyclic averaging combinations.

Cases 2, 3, and 4 are typical attempts to reduce the leakage by use of Burst Random excitation. The bursts lengths of 80%, 50% and 20%, respectively, span the range of practical burst excitation record lengths. The increase in coherence indicates that while leakage has been reduced, it has not been eliminated by reducing the burst length. Further reduction in burst length would seriously degrade the SNR, as well as, cause the test technique to approach impact testing with all its attendant problems. A different testing/averaging technique is required to eliminate leakage.

Cases 5 and 7 show the use of cyclic averaging with Random excitation. The reduction in leakage for these cases is comparable to the use of 80% Burst Random.

Cases 6 and 8 show the use of cyclic averaging with Burst Random excitation. For both cases, the burst length was chosen such that the burst continued 20% into the last block. The reduction in leakage is significant when compared to the previous cases, but is
Case 9 shows the use of cyclic averaging with Burst Random excitation, but this time the burst length is chosen as 80% of the total record length. This allows 1.2 blocks for the response to decay. In this case, the leakage is virtually eliminated. This reduction is not possible when using only burst excitation or cyclic averaging alone.
Figure 13. Case 3: Burst Random Excitation (50 %)

Figure 14. Case 4: Burst Random Excitation (20 %)

Figure 15. Case 5: Random Excitation with Hann Window, 4 Cyclic Averages

Figure 16. Case 6: Burst Random Excitation (80 %), 4 Cyclic Averages

Figure 17. Case 7: Random Excitation with Hann Window, 6 Cyclic Averages

Figure 18. Case 8: Burst Random Excitation (86 %), 6 Cyclic Averages
Table 2 is a summary of the FRF magnitude/multiple coherence for several peak frequencies for each case. Comparing the relative magnitude of the FRF for Cases 1 and 4 to Case 9, the error in magnitude is severe for the random excitation and significant for the burst random (20%). These two cases are representative of typical measurement conditions and the burst random is the best non-cyclic average result. The random excitation result averages approximately 70% magnitude error. The burst random, while much better showing 5% or less error for most peaks, exhibit about 30% magnitude error for the 77.5 Hz peak. When estimating modal parameters, the frequency and mode shape would probably be estimated reasonably, however, the damping and modal scaling would be distorted (over estimating damping and under estimating modal scaling). Using these results for model prediction or FE correction would bias the predicted results.

5. Conclusions

The most important conclusion that can be drawn from the results of this measurement exercise on a lightly damped mechanical system is that accurate data is an indirect function of measurement time or number of averages but is a direct function of measurement technique. The leakage problem associated with utilizing fast Fourier transform (FFT) methodology to estimate frequency response functions on a mechanical system with light damping is a serious problem that can be managed with proper measurement techniques, like cyclic averaging and burst random excitation. It is also important to note that while ordinary/multiple coherence can indicate a variety of input/output problems, a drop in the ordinary/multiple coherence function, at the same frequency as a lightly damped peak in the frequency response function, is often a direct indicator of a leakage problem. Frequently, comparisons are made between results obtained with narrowband (sinusoid) excitation and broadband (random) excitation when the ordinary/multiple coherence function clearly indicates a potential leakage problem. It is important that good measurement technique be an integral part of such comparisons.

6. References

### Table 1. Test Cases - DSP Parameters

<table>
<thead>
<tr>
<th>Case</th>
<th>Excitation</th>
<th>Window</th>
<th>Blocksize</th>
<th>$F_{\text{max}}$</th>
<th>$N_c$</th>
<th>$N_s$</th>
<th>$N_{\text{avg}}$</th>
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</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>Random</td>
<td>Hann</td>
<td>2048</td>
<td>512 Hz.</td>
<td>1</td>
<td>96</td>
<td>96</td>
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<tr>
<td>Case 2</td>
<td>Burst Random (80%)</td>
<td>Uniform</td>
<td>2048</td>
<td>512 Hz.</td>
<td>1</td>
<td>96</td>
<td>96</td>
</tr>
<tr>
<td>Case 3</td>
<td>Burst Random (50%)</td>
<td>Uniform</td>
<td>2048</td>
<td>512 Hz.</td>
<td>1</td>
<td>96</td>
<td>96</td>
</tr>
<tr>
<td>Case 4</td>
<td>Burst Random (20%)</td>
<td>Uniform</td>
<td>2048</td>
<td>512 Hz.</td>
<td>1</td>
<td>96</td>
<td>96</td>
</tr>
<tr>
<td>Case 5</td>
<td>Random</td>
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<td>512 Hz.</td>
<td>4</td>
<td>24</td>
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<td>Case 6</td>
<td>Burst Random (80%)</td>
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<td>96</td>
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<td>Case 7</td>
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<td>512 Hz.</td>
<td>6</td>
<td>16</td>
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<td>Case 8</td>
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<td>512 Hz.</td>
<td>6</td>
<td>16</td>
<td>96</td>
</tr>
<tr>
<td>Case 9</td>
<td>Burst Random (80%)</td>
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<td>2048</td>
<td>512 Hz.</td>
<td>6</td>
<td>16</td>
<td>96</td>
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### Table 2. Test Cases - Measurement Results

<table>
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<th>Case</th>
<th>24 Hz. Peak</th>
<th>55.5 Hz. Peak</th>
<th>77.5 Hz. Peak</th>
<th>162.5 Hz. Peak</th>
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<td>Coh.</td>
<td>FRF</td>
<td>Coh.</td>
<td>FRF</td>
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<td>0.0187</td>
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<tr>
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<td>0.9965</td>
<td>0.0170</td>
<td>0.9957</td>
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<tr>
<td>Case 9</td>
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<td>0.0173</td>
<td>0.9977</td>
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