

COMPUTERIZED DETERMINATION  
OF  
VIBRATION FREQUENCIES  
IN  
POWER TRAINS

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## I. INTRODUCTION

In the study of vibrations of operating equipment, one of the most difficult tasks facing the vibrations engineer is identification of the myriad of peaks which will be found in the power spectrum data on even the most simple types of equipment. Modern digital signal processing equipment has given us the ability to measure vibration spectra with great precision and frequency resolution. If this same equipment can be programmed to identify the various peaks in the measured spectra it will go a great way toward helping the vibrations engineer establish causal relationships between high vibration spectra and failure of mechanical components.

This thesis is the result of a feasibility study into such a program. The program described herein is in no way meant to be a finished product nor is it meant to cover all possibilities in terms of computerized identification of power train vibration components. It is, however, a meaningful first step in this process.

## II. BACKGROUND INFORMATION

At first glance one who is not familiar with the analysis of vibrations of rotating machines might think that identification of vibration spectra from a transmission or other piece of rotating equipment should be relatively easy. One would expect that a gear set would give off vibrations at a frequency equal to the number of teeth times the rotational speed. Any rotating shaft is of course going to show vibrations at the rotational speed caused by the inherent unbalance in the shaft. The equations for bearings are more difficult but they can be found in the literature [1]. If all of these frequencies are so easy to calculate, why do we need a computer program?

There are many answers to the above question. Probably the most significant is that we normally make spectrum measurements to determine the cause of an undesirable vibration or noise and not to determine if a particular harmonic of a frequency of a particular component shows up in the vibration or noise spectrum. While the equations required to calculate the frequencies produced by a power train component are fairly simple, the sheer number of possibilities makes the task of back calculating from a peak in the noise or vibration spectrum to a physical cause nearly impossible.

Another reason to have a computer program to identify spectral components is that all the frequencies are not the results of simple relationships between power train components. There also may be harmonics of the fundamental frequencies. If modulation is present, which it often is, there will also be sidebands of the fundamental frequencies and their harmonics. If the modulation is non-sinusoidal, which is again typically the case, there will also be harmonics of the sidebands. What all this means is that in many cases even a seemingly simple mechanical system can produce a very complicated vibration spectrum.

As a case in point consider the system shown in Figure 2.1. This system consists of a variable speed electric motor, a shaft with a flywheel supported in sleeve bearings and driven through a neoprene coupling. It is difficult to imagine a simpler system. We would expect a spectrum measurement on this system to show a peak corresponding to the running speed due to the unbalance in the shaft and perhaps some response at the electrical line frequency due to electric forces in the motor.

Figure 2.2 is an RPM spectral map made from data taken on the system of Figure 2.1. The RPM spectral map was created by making vibration power spectrum measurements while doing a slow speed sweep over the range from 3000 to 10000 RPM. A tachometer signal taken with the data determines the

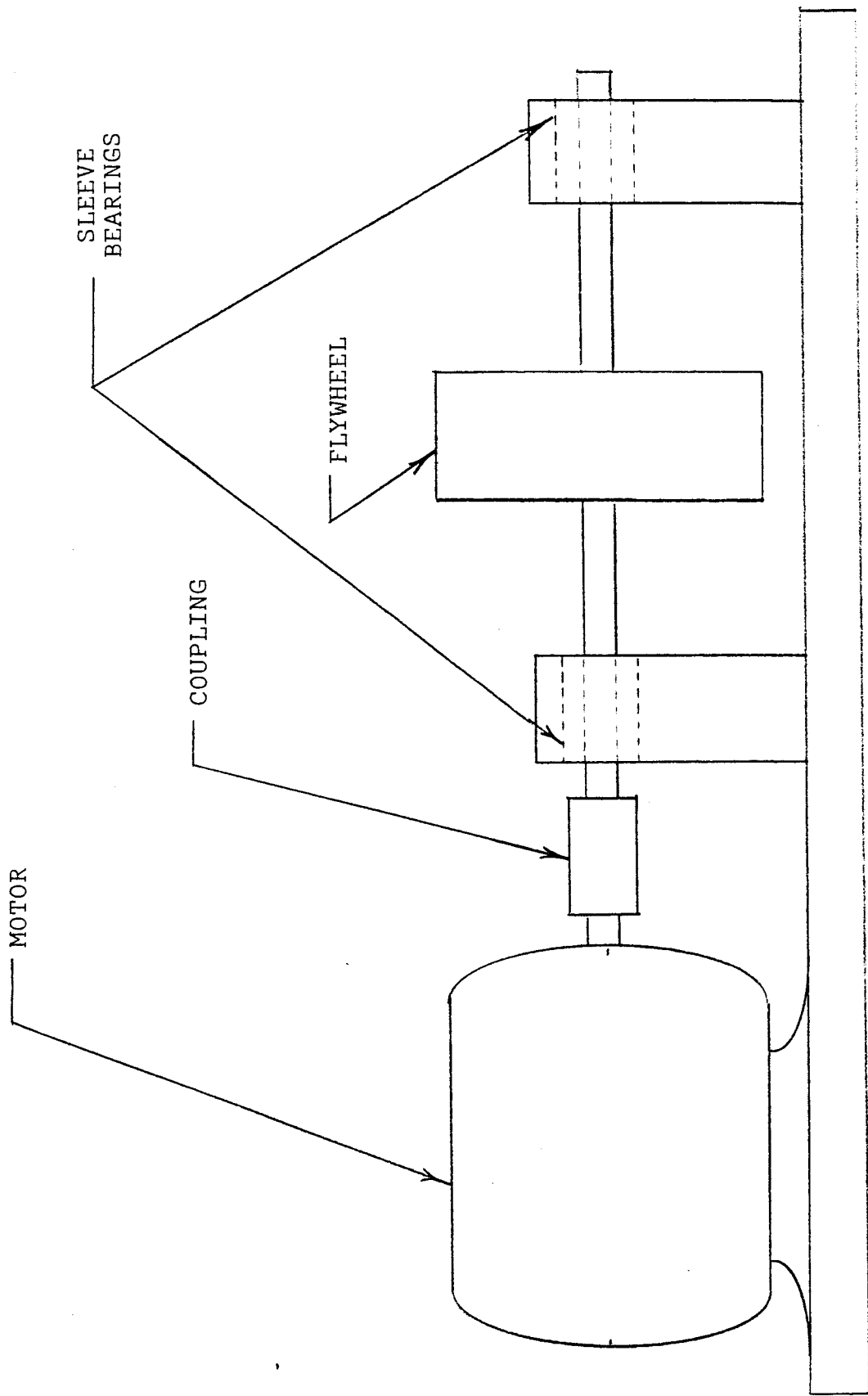
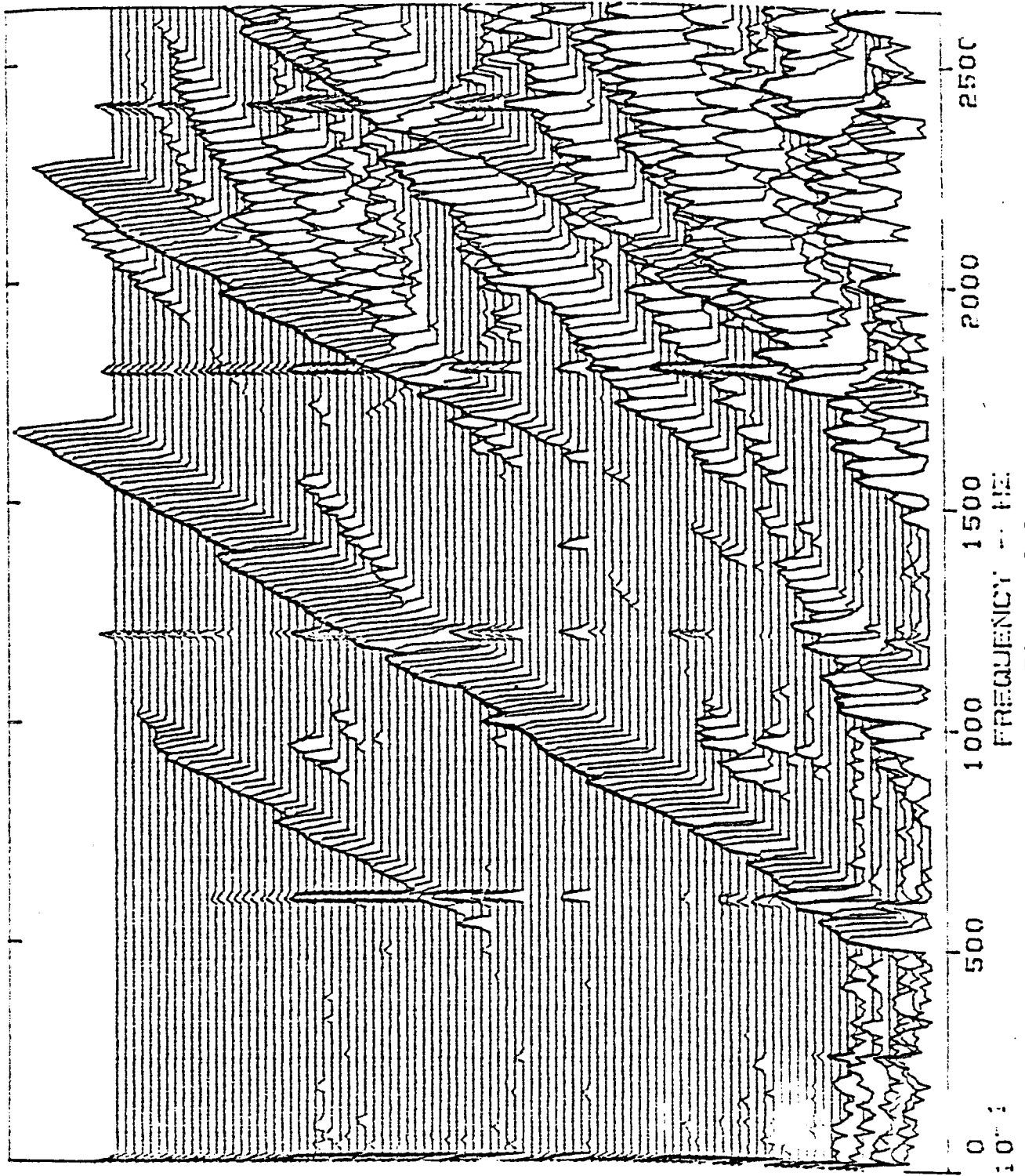


Figure 2.1



10000--

REVOLUTIONS / MIN

5000--

Figure 2.2  
RPM Spectral Map

vertical position of each power spectrum on the plot. On this type of plot vibrations which are related to the speed of the rotating device will show up as rays emanating from the zero frequency zero-speed origin while vibrations which are fixed in frequency show up as vertical lines.

As expected, we see a strong peak starting at about 50 HZ at 3000 RPM and continuing to about 165 HZ at 10,000 RPM which can be identified as the shaft speed. There is also some response at the electrical frequencies as expected. It is also obvious that there are a great number of peaks which do not seem to be simply related to rotational speed or frequency. Some of these peaks are caused by harmonics of the fundamental shaft speed vibration but the majority of these peaks are caused by modulation of the fundamental vibration by electrically induced vibration at the line frequency and its harmonics. For example two rows of peaks can be seen which parallel the running speed peaks. These are the upper and lower sidebands of the running speed as modulated by 60 HZ electrically induced vibration. (The topic of modulation will be covered in more detail in Section III). As we get higher harmonics of the running speed being modulated by higher harmonics of the line frequency, the picture becomes even more difficult to interpret. This is seen in the area from 150 to 250 HZ. It is not hard to imagine how difficult it would be to interpret a spectrum



measurement made on the above piece of equipment if it ran at fixed speed and we did not have the luxury of an RPM spectral map or a computer program.

### III. THEORY

In general, the equations used in the identification of frequencies produced by a power train are very elementary and will not be discussed in this section. These equations are listed in Appendix C for those who are interested. Two exceptions to the above are the equations related to amplitude modulation and bearing defects which are covered in the following paragraphs.

#### III.1 Amplitude Modulation Theory

Amplitude modulation is a phenomenon which occurs when one signal is multiplied by another signal. Modulation is most often encountered in communications where it is used to increase the efficiency of transmission of a signal. Because of this fact, most references on modulation are written with the communication industry in mind.

Modulation can also occur in rotating equipment such as gearboxes. Modulation in a gearbox, by itself, is neither an indication of poor health or good health of the gearbox. It is merely another piece of information which, if we understand it properly, can be used to relate a measured vibration signal to a physical phenomenon.

One mechanism by which amplitude modulation can occur in a gearbox is through a variation in the effective pitch

diameter of one or both of the gears, which is a function of the rotational position of the gear. The most obvious way this can occur is if the geometric center of the gear does not coincide with the center of the axis of rotation, either due to manufacturing defects or damage such as a bent shaft. It can also occur if worn bearings allow the shaft supporting the gear to move in a periodic fashion related to the rotation angle.

The effect of the above mechanism is that the gear teeth see a load which is comprised of some constant value plus a sinusoidally varying component with period equal to the period of rotation of the modulating shaft.

Consider the gear pair shown in Figure 3.1. For simplicity assume that gear 1 is a perfect gear and that gear 2 has a defect of the type described above. The vibration caused by the mating of these gears would be of the form:

$$V(t) = G \text{ COS } (n_1 \omega_1 t) L(t) \quad (3.1)$$

where:  $V(t)$  = vibration signal

$n_1$  = number of teeth on gear 1

$\omega_1$  = rotational speed of gear 1

$L(t)$  = load seen by gear teeth

$G$  = compliance coefficient

$L(t)$  can be given by:

$$L(t) = A + B \text{ COS } \omega_2 t \quad (3.2)$$

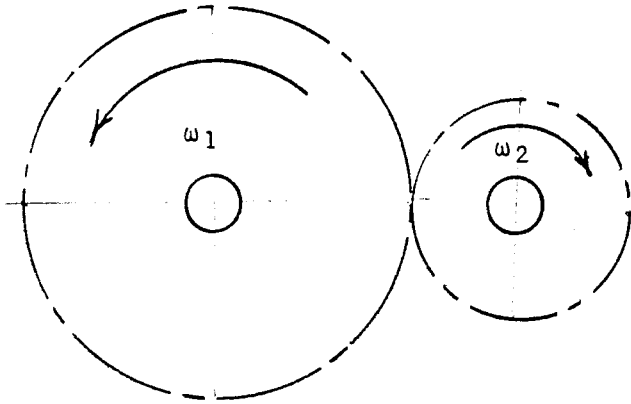


Figure 3.1

where A and B are constants.

Substituting equation 3.2 into equation 3.1 yields:

$$V(t) = GA \cos n_1 \omega_1 t + GB (\cos n_1 \omega_1 t) (\cos \omega_2 t) \quad (3.3)$$

or using trigonometric identities:

$$\begin{aligned} V(t) = GA \cos n_1 \omega_1 t + 1/2 GB \cos (n_1 \omega_1 - \omega_2) t \\ + 1/2 GB \cos (n_1 \omega_1 + \omega_2) t \end{aligned} \quad (3.4)$$

Fourier transforming equation 3.4 yields:

$$\begin{aligned} V(\omega) = GA\pi [\delta(\omega - n_1 \omega_1) + \delta(\omega + n_1 \omega_1)] \\ + G \frac{B\pi}{2} [\delta(\omega - (\omega_2 - n_1 \omega_1)) + \delta(\omega - (\omega_2 + n_1 \omega_1)) \\ + \delta(\omega - (-\omega_2 - n_1 \omega_1)) + \delta(\omega - (-\omega_2 + n_1 \omega_1))] \end{aligned} \quad (3.5)$$

$V(t)$  and the positive half spectrum of  $V(t)$  are shown in Figure 3.2.

Thus we see that amplitude modulation causes sidebands at plus and minus  $\omega_2$  from the fundamental frequency. If the modulating signal is not purely sinusoidal then sidebands would also be produced at plus and minus harmonics of the modulating signal. The same phenomenon will occur at harmonics of the fundamental frequency.

### III.2 Derivation of Bearing Equations

Given a ball bearing as shown in Figure 3.3 the equations for the fundamental frequencies produced by various defects can be derived as follows.

Assume no slippage:

The velocity of point  $P_1$ , which is the point of contact between the ball and the inner race, is just the velocity of point  $P_1$  on the inner race.

$$V_{P1} = R_{P1}\omega_i = (Pr - Rr \cos \theta)\omega_i \quad (3.6)$$

where:  $R_{P1}$  = radius to point  $P_1$  from bearing center

$\omega_i$  = rotational speed of inner race-RPM

$Pr$  = bearing pitch radius

$Rr$  = rolling element radius

$\theta$  = bearing contact angle

$\omega_o$  = rotational speed of outer race

Similarly:

$$V_{P2} = -R_{P2}\omega_o = -(Pr + Rr \cos \theta) \omega_o \quad (3.7)$$

Since the ball is a rigid body with center midway between  $P_1$  and  $P_2$  the velocity of the center of the ball is given by:

$$V_c = \frac{V_{P1} + V_{P2}}{2} = \frac{\omega_i(Pr - Rr \cos \theta) - \omega_o(Pr + Rr \cos \theta)}{2} \quad (3.8)$$

The rotational speed of the ball train or carrier is then given by:

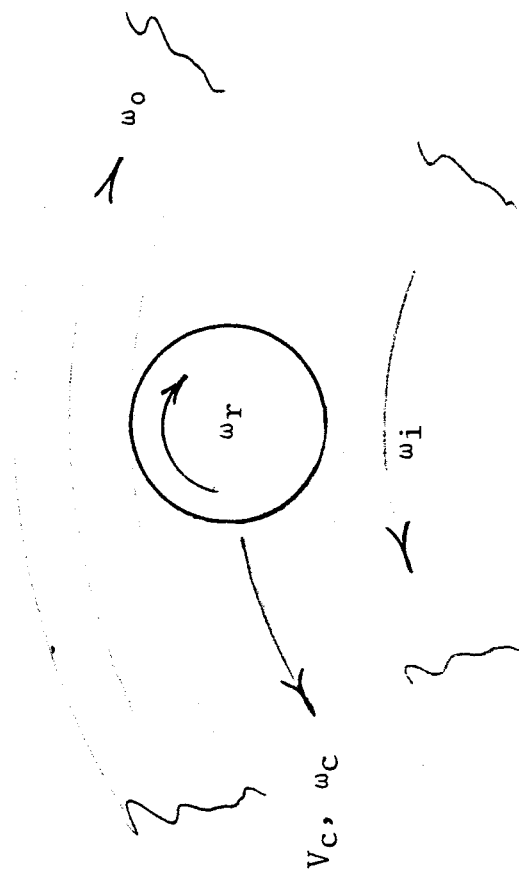
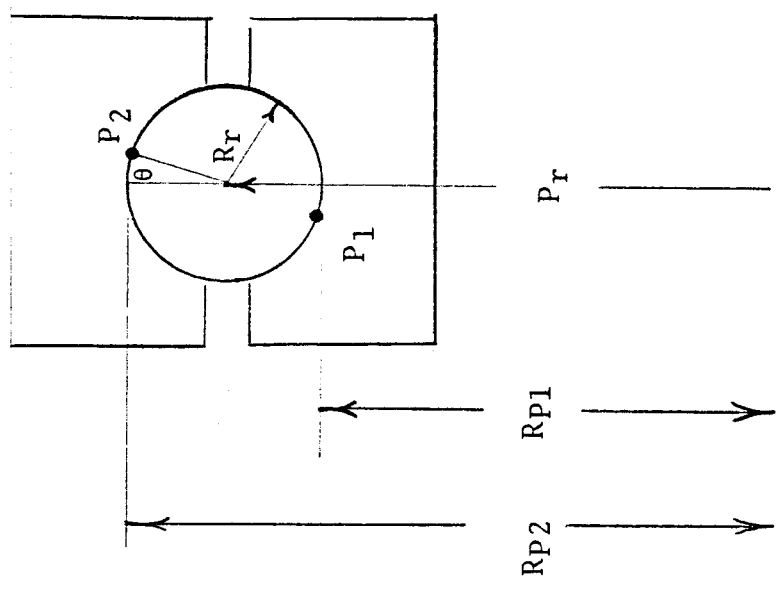


Figure 3.3

$$\omega_c = \frac{V_c}{Pr} = \frac{\omega_1(Pr - R_r \cos \theta) - \omega_o(Pr + R_r \cos \theta)}{2 Pr} \quad (3.9)$$

Substituting RPM for radian frequency, the frequency due to a defect on the inner race is then given by the number of rolling elements times the relative RPM between the carrier and the inner race:

$$f_i = \frac{n}{60} (N_i - N_c) \text{ Hz} \quad (3.10)$$

where  $n$  = number of rolling elements

$N_i$  = RPM of inner race

$N_o$  = RPM of outer race

or :

$$f_i = \frac{n}{60} \left[ N_i - \left( \frac{N_i(Pr - R_r \cos \theta) - N_o(Pr + R_r \cos \theta)}{2 Pr} \right) \right] \text{ Hz} \quad (3.11)$$

Similarly the fundamental frequency due to a defect on the outer race is given by:

$$f_o = \frac{n}{60} (N_i + N_o) \quad (3.12)$$

or :

$$f_o = \frac{n}{60} \left[ N_o + \left( \frac{N_i(Pr - R_r \cos \theta) - N_o(Pr + R_r \cos \theta)}{2 Pr} \right) \right] \text{ Hz}$$

The fundamental frequency of a defect on the rolling element can be calculated as the ratio of the rolling element radius to the inner or outer race radius time the relative velocity between the ball carrier and the appropriate race :



$$f_b = \frac{1}{30} \frac{(N_i - N_c) (P_r - R_r \cos \theta)}{R_r} \text{ Hz} \quad (3.14)$$

Note the factor of 2 correction required to account for the fact that the defect will produce a pulse each time it contacts the inner and outer race. This factor is sometimes not accounted for in published versions of these equations [2].

#### IV. GENERAL DESCRIPTION OF THE FREQUENCY IDENTIFICATION PROCESS

The frequency identification process makes use of the Hewlett-Packard 5451B Fourier Analyzer System. This system consists of a general purpose digital computer with special hardware and software to allow it to do signal measurement and processing.

The computer system itself is a general purpose high speed digital processor which can be programmed using various programming languages such as BASIC, FORTRAN, ALGOL, and ASSEMBLY language. In this "computer normal" mode, the system can be used for engineering calculations, accounting, inventory, process control, data sorting and processing, or any other function for which minicomputers are used. The addition of appropriate peripheral equipment enables its use as a preprocessor or postprocessor for a larger "master" computer, an intelligent terminal, or a time-shared computing facility.

When in the "Fourier Analyzer" mode, a special system program acts as an executive for a software-controlled signal analyzer. An analog-to-digital conversion peripheral is activated by a keyboard pushbutton to sample an analog signal and store a series of digital values which represent the signal amplitude at successive equally spaced sampling instants. At this point the information in the signal is

immune to errors in calibration, and are subject only to the very low truncation and roundoff errors inherent in digital computation. The operator may select one through four channels to be sampled simultaneously.

The sampled signal is then displayed on a CRT display unit. The analysis of the signal is done by invoking software subroutines by a keyboard stroke. The standard analysis routines are:

- Digital Fast Fourier Transform (forward and inverse)
- Auto Correlation, Cross Correlation
- Convolution, Digital Filtering
- Auto Power Spectrum, Cross Power Spectrum
- Transfer Function, Coherence Function
- Histogram

The system also contains routines for arithmetic manipulation of the signals:

- Integration
- Differentiation
- Multiplication of two signals, or a signal by a complex constant
- Division of one signal by another, or of a signal by a complex constant
- Subtraction of signals
- Conjugate multiplication for complex-valued signals.

